Transparent Boundary Conditions for Wave Propagation in Fractal Trees

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In this work we consider the problem of sound propagation in a bronchial network of the human lung. Asymptotically, this phenomenon is modelled by a weighted wave equation posed on a fractal (i.e. self-similar) 1D tree [1, 2]. The principal difficulty in the numerical resolution of this problem stems from the structural infiniteness of the domain, on one hand, and from the presence of the reflections from the 'infinite' boundary and the nodes of the tree, on the other hand. To overcome this problem, we propose to use approximate transparent boundary conditions, based on an approximation of the DtN (Dirichlet-to-Neumann) operator. The latter is a convolution operator, whose convolution kernel is not known in a computable form ; nonetheless, it is possible to compute its symbol, which satisfies a certain non-linear equation. Moreover, it is known that this symbol can be represented as a series of simple fractions, whose poles and residues are related respectively to the eigenvalues and normal traces of eigenfunctions of a weighted Laplacian on such a fractal tree.
Based on this information about the symbol of the DtN, we propose two approaches for approximate transparent boundary conditions : the first one is a convolution quadrature [3, 4] based approach, while the second one is based on a local approximation of the DtN map, in the spirit of [5], which, in turn, stems from truncating the series of simple fractions representing the symbol of the DtN.
We present error and complexity estimates for both approaches, and conclude the discussion with numerical experiments.

Références

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