Simulation of an Homogeneous Relaxation Model for a three-phase mixture

Jean BUSSAC, Laboratoire de Mathématiques Jean Leray - Nantes

Multiphase flow studies have a wide range of applications, especially in the nuclear framework, and homogeneous models have been recently considered for that purpose, see for example [2], [4] and references therein.

Here we consider the Homogeneous Relaxation Model (HRM) presented in [3], where all phases share the same velocity and the fluid is out of equilibrium. This model is based on compressible Euler equations on which advection equations on fractions Y are added with source terms.

Firstly, the convective part has already been studied in [3], and has the Euler system structure. We can demonstrate that the system admits a Lax entropy and is hyberbolic.

Secondly, source terms have to be chosen according to the entropy growth. Two types are considered and compared : the classical form $|Y - Y_{eq}|$, where Y_{eq} stands for the equilibrium fractions, and a second one $\nabla_Y \sigma$ presented in [3], with σ the mixture intensive entropy.
The first one has some advantages : exact solving, simple ODE trajectories. However, finding the equilibrium fractions can be really difficult depending of the EOS, and fraction relaxation times are all the same. The second one allows us to use different time scales, and does not need to compute the equilibrium. On the other side, ODE analysis is difficult and its trajectories are much more complex.

For the numerical applications, we mainly use a stiffened gas Equation Of State (EOS), which represents a good compromise between realistic modelisation and complexity. Following [4], we also consider the NASG-CK law for the liquid in some parts of our work.

Using a fractionnal-step method, we first solve the convective part with Finite Volumes methods (VFRoe-ncv and relaxation schemes), whereas source terms contribution is taken into account solving an ODE.

The first type of source term can be used computing the equilibrium with a non-linear solver, for example a Broyden algorithm, see [4]. Another solution, in the case of three stiffened gas, is to show that the mixture pressure follows a stiffened gas EOS itself, see [1] and the second appendix of [4].
The second type of source term requires an implicit ODE solver and a non-linear solver. Coupling Euler's method with a Broyden algorithm allows us to solve the ODE.

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[2] O. Hurisse. *Application of an homogeneous model to simulate the heating of two-phase flows.* International Journal on Finite Volumes, **11**, http–www, 2014.

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[4] L. Quibel. *Simulation of water-vapor two-phase flows with non-condensable gas.* Ph.D. thesis, Université de Strasbourg, 2020.

Contact : `jean.bussac@univ-nantes.fr`