Taux de décroissance exponentielle pour flots gradients dégénérés soumis à une condition dexcitation persistente

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In this talk we will present some recent results [2] on the worst rate of exponential decay for systems of the type
\[ \dot{x}(t) = -c(t)c(t)\top x(t), \quad x(t) \in \mathbb{R}^n, \quad (DGF) \]
where the signal \( c : [0, +\infty] \to \mathbb{R}^n \) is square integrable and verifies the persistent excitation condition. That is, there exist constants \( a, b, T > 0 \) such that
\[ \forall t \geq 0, \quad a \text{Id}_n \leq \int_t^{t+T} c(s)c(s)\top ds \leq b \text{Id}_n. \quad (PE) \]
These dynamics appears in the context of adaptive control and identification of parameters, and are usually referred to as degenerate gradient flow systems. It is well-known that the persistent excitation condition is equivalent to the global exponential stability of the system.

The rate of exponential decay for (DGF) is the positive quantity \( R(c) \) defined by
\[ R(c) := -\limsup_{t \to +\infty} \frac{\log \|\Phi_c(t,0)\|}{t}, \]
where \( \Phi_c(t,t_0) \) denotes the flow of (DGF) from \( t_0 \) to \( t \). The worst rate of exponential decay for (DGF) is the positive quantity \( R(a,b,T,n) \) obtained as the infimum of \( R(c) \) as the signal \( c \) runs over all square-integrable functions satisfying (PE). We observe that lower bounds for \( R(a,b,T,n) \) of the form
\[ R(a,b,T,n) \geq \frac{Ca}{(1+nb^2)T}, \]
are well-known [1].

Our main result is then the following, which shows that for \( n \) fixed the known lower bounds are indeed optimal.

**Théorème 1.** There exists \( C_0 > 0 \) such that for every \( n \in \mathbb{N}, T > 0, \) and \( 0 < a \leq b \) it holds
\[ R(a,b,T,n) \leq \frac{C_0a}{(1+b^2)T}. \]

The proof is based on recasting the computation of \( R(a,b,T,n) \) as an optimal control problem, which is then carefully analysed to yield the result.
As a byproduct of our technique, we also prove necessary conditions for the exponential converges of systems of the form (DGF), under more general persistent excitation conditions.


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