Taux de décroissance exponentielle pour flots gradients dégénérés soumis à une condition dexcitation persistente

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In this talk we will present some recent results [2] on the worst rate of exponential decay for systems of the type

$$\dot{x}(t) = -c(t)c(t)^\top x(t), \qquad x(t) \in \mathbb{R}^n, \tag{DGF}$$

where the signal $c : [0, +\infty] \to \mathbb{R}^n$ is square integrable and verifies the *persistent excitation* condition. That is, there exist constants $a, b, T > 0$ such that

$$\forall t \geq 0, \quad a\,\mathrm{Id}_n \leq \int_t^{t+T} c(s)c(s)^\top\,ds \leq b\,\mathrm{Id}_n\,. \tag{PE}$$

These dynamics appears in the context of adaptive control and identification of parameters, and are usually referred to as *degenerate gradient flow systems*. It is well-known that the persistent excitation condition is equivalent to the global exponential stability of the system.
The rate of exponential decay for (DGF) is the positive quantity $R(c)$ defined by

$$R(c) := -\limsup_{t\to+\infty} \frac{\log \|\Phi_c(t,0)\|}{t},$$

where $\Phi_c(t, t_0)$ denotes the flow of (DGF) from t_0 to t. The *worst rate of exponential decay* for (DGF) is the positive quantity $R(a, b, T, n)$ obtained as the infimum of $R(c)$ as the signal c runs over all square-integrable functions satisfying (PE). We observe that lower bounds for $R(a, b, T, n)$ of the form

$$R(a, b, T, n) \geq \frac{Ca}{(1 + nb^2)T},$$

are well-known [1].
Our main result is then the following, which shows that for n fixed the known lower bounds are indeed optimal.

Théorème 1. *There exists $C_0 > 0$ such that for every $n \in \mathbb{N}$, $T > 0$, and $0 < a \leq b$ it holds*

$$R(a, b, T, n) \leq \frac{C_0 a}{(1 + b^2)T}.$$

The proof is based on recasting the computation of $R(a, b, T, n)$ as an optimal control problem, which is then carefully analysed to yield the result.
As a byproduct of our technique, we also prove necessary conditions for the exponential converges of systems of the form (DGF), under more general persistent excitation conditions.

[1] S. Andersson, P. Krishnaprasad. *Degenerate gradient flows : A comparison study of convergence rate estimates.* In *Proceedings of the 41st IEEE Conference on Decision and Control, 2002.*, vol. 4, pp. 4712–4717. IEEE. doi :10.1109/CDC.2002.1185122.

[2] Y. Chitour, P. Mason, D. Prandi. *Worst exponential decay rate for degenerate gradient flows subject to persistent excitation.* To appear on SIAM Journal on Control and Optimization (SICON), 2021.

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