On second-order well-balanced Lagrange-projection schemes for shallow water Exner equations

Christophe CHALONS, Laboratoire de Mathématiques de Versailles - Versailles, France
Alessia DEL GROSSO, Laboratoire de Mathématiques de Versailles - Versailles, France

Our work is devoted to the development and implementation of second-order well-balanced Lagrange-projection numerical methods applied to the shallow water Exner system. The Lagrange-Projection formalism entails a decomposition of the mathematical model into two different systems : the acoustic/Lagrangian one, which takes into account the compressibility effects of the flow, and the transport/projection system for the transport phenomena [Morales de Luna, Castro Díaz, Chalons (2020)]. As far as the mathematical model is concerned, we consider the well-known shallow water system coupled with the so-called Exner equation, used to simulate the bedload sediment transport due to the mechanical action of the flow [Cordier, Le, Morales de Luna (2011)]. In particular the Grass closure formula is taken into account to model the solid transport discharge contributions. It is known that it is not a trivial task to numerically simulate the resulting shallow water Exner model, as a decoupled or weakly coupled method could lead to the presence of spurious oscillations in the numerical outputs. Moreover, when considering the Lagrangian-projection formalism, while it is clear how to decompose the shallow water system, this is not true when it comes to the Exner equation. For this reason we investigate different possible numerical strategies. The methods and their second-order accurate extensions are designed in such a way to satisfy the well-balanced property, namely the ability of the scheme of preserving the stationary solutions of the model.

Contact : alessia.del-grosso@ens.uvsq.fr