A multigrid solver for the M_1 model for radiative transfer

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The problem of radiative transfer describes the interaction between light and matter, therefore it appears in many physical situations, such as astrophysical systems, laser-matter interactions, or atmospheric physics [2]. Instead of solving a complex equation in a seven dimensional space, one can use a moment model by averaging over the direction of propagation to follow the radiative energy, flux, pressure, etc in a five dimensional space. By using a closure relation expressing the radiative pressure as a function of radiative energy and flux, one can derive the M_1 model [3].
This model being hyperbolic, it can be discretized with an HLL solver. The coupling to hydrodynamics, needed for physical applications, requires the use of an implicit solver with a large time step. Indeed, the time step required by an explicit solver would be limited by the speed of light for the radiative transfer, whereas it is limited by the velocity of the fluid for the hydrodynamics [4].
Because the system to be solved is non-linear, we use the Jacobi method presented in [6]. It has been shown that this iterative method always preserves the admissible states, such as positive radiative energy.
To decrease the number of iterations needed for the solver to converge, and therefore to decrease the computational cost, we use a geometric multigrid algorithm (e.g. [1]). The main idea of this method is to project the problem on a coarse grid, where computations are easier. The solution computed on this coarse grid is then interpolated on the original grid. This process can be applied recursively, until there is only a few unknowns to be solved. Unfortunately, this method is not designed to preserve the admissible states. To tackle this issue, we introduce a pseudo-time time such that the solution of the problem on the coarse grid is the steady state of a differential equation in pseudo-time [5]. A well-chosen discretization of this equation allows us to preserve that admissible states. We present preliminary results showing the decrease of the number of iterations and computational cost as a function of the number of multigrid levels used in the method.

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