Double-scale diffusive wave model dedicated to spatial river observation and associated covariance kernels for variational data assimilation

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The spatial altimetry provides an important amount of water surface height data from multi-missions satellites (especially Jason-3, Sentinel-3A/B and the forthcoming NASA-CNES SWOT mission). To exploit at best the potential of spatial altimetry, the present study proposes the derivation of a model adapted to spatial observations scale : a diffusive-wave type model that takes into account both the scale of the continuum mechanics and of the altimetric observations scale (see [4]).
Moreover, Green-like kernels can be employed to derived covariance operators. Therefore, they may provide an approximation of the covariance kernel of the background error in Variational Data Assimilation (VDA) processes. In data assimilation, the estimation of the background error covariance is a classical and still open topic (see e.g. [1]). In this context, the VDA approach is developed to infer the bathymetry in the river flow models, here the Saint-Venant equations. Following the derivation of the aforementioned original two-scale flow model, we present the derivation of a Green-like kernel and of the consequent covariance kernels. These covariance kernels are used to model the covariance kernel of the background error for the bathymetry (see [3]). These physically-based covariances take into account more dynamics than the empirical covariance kernels classically-used. Moreover, it provides physically-consistent parameters.
The two parts of this study are applied on a real-like dataset derived from the Rio Negro River (Amazon basin). The diffusive wave equation adapted to the observations scale provides more accurate results than the classical version. It is especially true for the estimation of the discharge because the slope estimation computed from the observations scale water surface elevation takes into account variations inner to the observations scale. The presented method to estimate the background error covariance is investigated by comparing the physically-derived and classical kernels, such as the decreasing exponential or the Gaussian, with physically-consistent parameters with classical kernels, such as the decreasing exponential with arbitrary parameters (see e.g. [2]). The physically-derived and physically-consistent exponential kernels provide better accuracy and faster convergence than the classical kernels, especially in the first iterations of the VDA optimization process.

[1] R. N. Bannister. *A review of forecast error covariance statistics in atmospheric variational data assimilation. i : Characteristics and measurements of forecast error covariances.* Quarterly Journal of the Royal Meteorological Society, **134(637)**, 1951–1970, 2008. doi :10.1002/qj.339.

[2] K. Larnier, J. Monnier, P.-A. Garambois, J. Verley. *River discharge and bathymetry estimations from SWOT altimetry measurements.* Inverse Problems Sc. Eng., 2020.

[3] T. Malou, J. Monnier. *Covariance kernels investigation from diffusive wave equations for data assimilation in hydrology.* in prep., 2021.

[4] T. Malou, J. Monnier. *Double scale diffusive wave equation dedicated to spatial rivers observations.* in prep., 2021.

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