Innovative numerical schemes for 2D supersonic aerodynamics on hybrid mesh

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After a break of few decades, certain interest re-emerges for hypersonic aircraft design largely because of the availability of numerical simulations for multi-dimensional aerodynamic flows. The underlying physical model is the Navier-Stokes (N-S) equations describing the conservation laws of mass, momentum and energy of a viscous and heat-conductive compressible fluid. The physical variables of the flow undergo important and complex spatial and temporal variations. Hence, robust and accurate numerical methods are mandatory to capture such flows.
Historical numerical methods rely on finite volume (FV) approaches that naturally capture shocks using approximate Riemann solvers (RS). One common feature of most existing RS is that they were developed from the Eulerian point of view. It is nonetheless possible to adopt a different approach : for instance [2] proposes a general methodology to construct positivity preserving and entropy stable approximate RS for Lagrangian and Eulerian systems of conservation laws relying onto the Lagrange-Euler mapping. This talk presents the development of FV methods to solve the non-viscous and non-conductive part of the N-S equations, i.e. the two-dimensional Euler equations, extending the work presented in [1]. It focuses on unstructured meshes and use the Lagrangian framework as a stepping stone to derive an approximate RS in the Eulerian framework. On the ground of this, direct estimations of ordered wave speeds can be derived naturally, and the solver inherits the positivity-preserving properties of the Lagrangian one following the methodology initially introduced in the pioneering work [2].
Moreover, we propose a second approximate RS that will be referred as **multi-point** solver. It is based on the development of a multidimensional approximation of the numerical fluxes using nodal velocity solvers akin not only to those developed in the context of Lagrangian cell-centered FV schemes [3] but also to those developed recently for Eulerian gas dynamics [4]. We emphasize that the latter work employed an HLLC-like approximate RS for which the wave speeds ordering is not guaranteed. In the present work, the proposed approximate RS cures this flaw. Particular care will be taken once again in the study of positivity preserving property of the numerical method. Several numerical experiments utilizing well known test cases show that the proposed multi-point solver is much less sensitive to the carbuncle instability than classical approximate RS. To further improve accuracy, the scheme is extended to second-order in time and space. Several preliminary numerical studies and simulations will be presented assessing the interest of this novel approach.

[1] A. Chan, G. Gallice, R. Loubère, P.-H. Maire. *Positivity preserving and entropy consistent approximate Riemann solvers dedicated to the high-order MOOD-based finite volume discretization of Lagrangian and Eulerian gas dynamics.* Computers and Fluids, 2021. Submitted.

[2] G. Gallice. *Positive and entropy stable Godunov-type schemes for gas dynamics and MHD equations in Lagrangian or Eulerian coordinates.* Numer. Math., **94(4)**, 2003.

[3] P.-H. Maire, R. Abgrall, J. Breil, J. Ovadia. *A cell-centered Lagrangian scheme for two-dimensional com- pressible flow.* SIAM J. Sci. Computing, **29**, 2007.

[4] Z. Shen, W. Yan, G. Yuan. *A robust and contact resolving Riemann solver on unstructured mesh, Part I, Euler method.* Journal of Computational Physics, **268**, 2014.

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