The Stochastic Zakharov system in dimension 1

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The Zakharov system is a simplified model for the description of Langmuir oscillations in a ionized plasma (for example the ionosphere). Langmuir waves are rapid oscillations of the electron density. We are interested in the stochastic Zakharov system, with a damping term :

$$\begin{cases} i\partial_t u = -\partial_x^2 u + nu \\ \varepsilon^2\partial_t^2 n + \alpha\varepsilon\partial_t n = \partial_x^2(n + |u|^2) + \phi\frac{dW(t)}{dt} \end{cases} \tag{1}$$

where ϕ is a regularizing operator, and $\frac{dW(t)}{dt}$ is a space-time white noise. The function u represents the envelope of the electric field, and n denotes the deviation of the ion density from its mean mode. The term ε^2 refers to the inverse of the ion speed. The stochastic perturbation of this system can be understood as random external fluctuations, as for instance thermal fluctuations.
The deterministic version of this system (without damping term) was studied first by Sulem and Sulem in [5] where they proved global well-posedness in dimension 1 for regular initial data. More refinements were made by Added and Added in [1] to get global well-posedness in dimension 2 for small initial data. We can also cite [3] by Ginibre, Tsutsumi and Velo which proved local well-posedness in the energy space. The limit $\varepsilon \to 0$ of this system is studied for example in [2] where Added and Added showed the convergence to a cubic Nonlinear Schrödinger equation. In [4], Masmoudi and Nakanishi have provided a simpler proof of this convergence. We adapted the work of [5] to show almost sure global well-posedness in dimension 1, and we added a damping term to prove that for regular initial data, the solution u^ε of the stochastic Zakharov system converges in law to a stochastic Nonlinear Schrödinger equation :

$$i\partial_t u = -\partial_x^2 u - |u|^2 u + u\psi \circ \frac{dW(t)}{dt}. \tag{2}$$

To study this stochastic problem, we have to place ourselves in the context of approximation-diffusion, because the process (n^ε) has a non-zero time correlation length and we want it to converge to a white noise. The main difficulty comes from the fact that the energy of our system

$$H(u,n) = \|\partial_x u\|_{L^2}^2 + \frac{1}{2}\left(\|n\|_{L^2}^2 + \|\varepsilon\partial_t\partial_x^{-1} n\|_{L^2}^2\right) + \int_{\mathbb{R}} n|u|^2 dx \tag{3}$$

is no longer preserved, and more importantly it has a singular evolution as ε goes to 0. Thus we can not just adapt the work in [2], and we need to rescale our system to use the Perturbed Test Function method in order to correct the infinitesimal generator of $(u^\varepsilon, n^\varepsilon)$ and pass to the limit in the martingale problem.

[1] H. Added, S. Added. *Existence globale de solutions fortes pour les équations de la turbulence de Langmuir en dimension* 2. C. R. Acad. Sci. Paris Sér. I Math., **299(12)**, 551–554, 1984.

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[3] J. Ginibre, Y. Tsutsumi, G. Velo. *On the Cauchy problem for the Zakharov system.* J. Funct. Anal., **151(2)**, 384–436, 1997.

[4] N. Masmoudi, K. Nakanishi. *Energy convergence for singular limits of Zakharov type systems.* Invent. Math., **172(3)**, 535–583, 2008. doi :10.1007/s00222-008-0110-5.

[5] C. Sulem, P.-L. Sulem. *Quelques résultats de régularité pour les équations de la turbulence de Langmuir.* C. R. Acad. Sci. Paris Sér. A-B, **289(3)**, A173–A176, 1979.

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