The extension of the FVC scheme for two dimensional shallow water flows on unstructured triangular meshes

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The Saint-Venant equations are widely used to model tides, dam-break, storm, tsunamis, and generally the various geophysical shallow-water flows [5, 6]. This hyperbolic system of balance laws was introduced in [7] and it has been very commonly used in several works [1, 2, 3] when completed with appropriate terms. Solving these models numerically presents a challenge due to their nonlinear structure, irregular bathymetry, the additional source terms and also complex geometry.

We consider in this work the numerical resolution of the two dimensional version of this system with variable topographic source term on unstructured meshes by a new finite volume approach. We first present a simple and accurate homogeneous solver issued from a finite volume characteristics (FVC) scheme, that was introduced in the preliminary works [4, 8]. Then, we introduce a generalization of this scheme by preserving the positivity properties of the homogeneous solver and lead to a well-balanced scheme satisfying the steady-state condition of still water.
Finally, the proposed finite volume method is verified on several benchmark tests and shows good agreement with analytical solutions, moreover it gives a noticeable accuracy improvement compared to the original approach.

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