Model order reduction methods for Lindblad type equations

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We study the *cat qubit* proposal for the physical implementation of a quantum computer, where each qubit is encoded into the state of a quantum harmonic oscillator [2]. Assessing its suitability – e.g. the provided protection of the encoded logical information from external perturbations, the experimental parameter range to achieve this protection, etc. – relies on our ability to simulate this infinite-dimensional system in practically relevant settings. The simplest model of a cat-qubit is given by the following Lindblad type equation on $\mathcal{H} = L^2(\mathbf{R}, \mathbf{C})$ [2] :

$$\frac{d}{dt}\rho = D[\boldsymbol{a}^2 - \alpha^2](\rho) \tag{1}$$

where ρ is the *density operator* describing the system, *i.e.* a hermitian, positive semi-definite operator with unit trace on $\mathcal{H}$, $\alpha \in \mathbf{C}$ is a constant parameter, $\boldsymbol{a} = \frac{1}{\sqrt{2}}(x + \partial_x)$ is the *annihilation* operator of the harmonic oscillator and $D[L](\rho) := L\rho L^\dagger - \frac{1}{2}\left(L^\dagger L\rho + \rho L^\dagger L\right)$ for any (unbounded) operator L.
We will focus on the numerical simulation of (1), enriched with additional terms accounting for external pertubations, the effect of which we want to quantify. The main challenges are the infinite dimension of $\mathcal{H}$ and the preservation of the geometric properties of density operators : $\rho = \rho^\dagger \geq 0$, $Tr(\rho) = 1$.

The usual strategy for the simulation of a Lindblad type equation is a Galerkin projection onto the space spanned by a finite truncation of the *Fock basis*, an orthonormal basis of L^2 formed by the eigenstates of the harmonic oscillator hamiltonian $H = \boldsymbol{a}^\dagger\boldsymbol{a} + \frac{1}{2}I$. The number N of Fock states necessary to accurately represent the solution of (1) is, however, typically much larger than $|\alpha|^2$. This scaling prevents the method from extending to more complex situations like multi-qubits dynamics, which involve density operators on the tensor product $\mathcal{H}^{\otimes m}$, where $m \geq 1$ denotes the number of qubits considered. Indeed, in the range $|\alpha|^2 \in [2, 10]$ used in experiments [2], the density operator associated with m cat-qubits with a typical Fock truncation of $N = 100 \gg 10$ is represented by a matrix of size $N^m = 10^{2m}$ that quickly leads to untractable computations.
The recent work [1] introduced a so-called *shifted Fock basis* taylored to the simulation of (1), relying on the fact that we know the steady states of the unperturbed equation (1) and that the additionnal perturbation terms they consider are polynomials in $\boldsymbol{a}$ and $\boldsymbol{a}^\dagger$. We alternately propose to extract an efficient simulation basis from a few reference simulations by a systematic model order reduction strategy adapted to Lindblad type equations on density operators. We present numerical evidence that this strategy yields numerical results with an accuracy and a marginal cost in the online stage similar to that of simulations using the shifted Fock basis. This paves the way to an efficient numerical approach for more complex settings that are being addressed in ongoing works [3].

This is joint work with Claude Le Bris (Ponts ParisTech, Inria) and Pierre Rouchon (Mines ParisTech, Inria).

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[2] M. Mirrahimi, et al. *Dynamically protected cat-qubits : a new paradigm for universal quantum computation.* New Journal of Physics, 2014.

[3] L.-A. Sellem. *Mathematical methods for simulation, estimation and control of open quantum systems.* Ph.D. thesis, Mines ParisTech, Inria, In preparation.

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