Spectrum of the incompressible viscous Rayleigh-Taylor system

Olivier LAFITTE, LAGA - Université Sorbone Paris Nord
Tien-Tai NGUYEN, LAGA - Université Sorbone Paris Nord

The linear instability study of the viscous Rayleigh-Taylor model in the neighborhood of a laminar solution $(\rho_0(x_3), \vec{0}, p_0(x_3))$ of the incompressible Navier-Stokes system of equations in $\mathbf{R}^3$ with a gravitational field $-g\vec{e_3}$ amounts to the study of the following ordinary differential equation of order 4 :

$$-\lambda^2[\rho_0 k^2\phi - (\rho_0\phi')'] = \lambda\mu(\phi^{(4)} - 2k^2\phi'' + k^4\phi) - gk^2\rho_0'\phi, \tag{1}$$

where λ is the desired growth rate in time, k is the wave number transverse to the density profile. We first study the toy model $\rho_0(x_3) = \rho_+ 1_{\{x_3>0\}} + \rho_- 1_{\{x_3<0\}}$, with $\rho_\pm$ being positive constants (note that this profile is not smooth). We then derive the unique growth rate obtained for the toy model, hence generalize the well known inviscid Rayleigh-Taylor study between two immiscible fluids. Secondly, we consider two types of increasing profile, that are ρ_0' being compactly supported and being positive everywhere. We prove that, contrary to the toy model, and in accordance with the results of Guo and Hwang, Helffer and Lafitte for the incompressible Euler system of equation with a gravitational field, there is an infinite sequence of non trivial solutions (λ_n, ϕ_n) of (1), with $\lambda_n \to 0$ when $n \to +\infty$ and $\phi_n \in H^4(\mathbf{R})$.

Références

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Contact : tientai.nguyen@math.univ-paris13.fr