Analysis of a Multi-layer Shallow Water Model

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The Shallow-Water system is a well known and efficient approximation of the Navier-Stokes System with low computational cost, but its range of application is limited and does not allow access to the vertical profile of the horizontal velocity. In the classical Multi-Layer Shallow Water models, the main idea is to recover information on the variations of the horizontal velocity profile along the vertical axis by a Galerkin-type approximation

$$u(t,x,z) \mapsto \sum_{i=1}^{N} u_\alpha(t,x) \mathbf{1}_{z \in [z_{\alpha-1/2}, z_{\alpha+1/2}]},$$

in the fluid layers without breaking free from the physical restrictions of the Shallow Water problem. Following this approach, the first multi-layer (single fluid) model was introduced [1]. It is known that the multi-layer system in [1] (without mass exchange) loses the property of hyperbolicity. For this reason, E. Audusse, M.O. Bristeau, B. Perthame and J. Sainte-Marie introduced in [2] another multi-layer model based on the same vertical discretization, but taking into consideration a mass exchange term between the fluid layers.

The objective of our work, is to study the well-posedness of the Multi-layer Shallow Water system given in [2]. First, we furnish a full characterization of the hyperbolicity zone for the three layer model by the help of some tools in Algebraic Geometry [4],[3]. Second, we extend our analysis into an arbitrary number of layers. Furthermore, we consider a transformation of the Hydrostatic Euler system as in [5]. We aim to study the eigenvalues of the new system with operator matrix.

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