

Optimal decay of the parabolic semigroup in stochastic homogenization for correlated coefficient fields

Nicolas CLOZEAU, Laboratoire Jacques-Louis Lions - Paris

In this talk, we study the large scale behavior of elliptic systems with stationary and ergodic random coefficients that have only slowly decaying correlations. To this aim we analyze the so-called corrector equation, a degenerate elliptic equation posed in the probability space. In this work (see [2]), we use a parabolic approach and optimally quantify the time decay of the semigroup. While previous work in this direction was so far restricted to stationary random coefficients a with a finite range of dependence (that is, $a|_{\mathcal{U}}$ and $a|_{\mathcal{V}}$ are independent as soon as $\text{dist}(\mathcal{U}, \mathcal{V}) \geq 1$), see [1, 3], we propose in this contribution an analysis where the coefficient have slowly decaying correlations. As an example, we can treat Gaussian type coefficients (that is, of the form $a = A(g)$ with g Gaussian and A smooth) for which the covariance function decays with an algebraic rate (possibly very slow). The key objects in the analysis are functional inequalities (which can be seen as Poincaré's inequalities in the probability space, in a multi-scale level) which quantify the ergodicity. I will split the discussion in three parts. First, I will present the parabolic problem solved by the semigroup and make clear the relationship with the corrector equation. Then, I will explain how functional inequalities allow us to optimally quantify the time decay of the semigroup and, for this, I will investigate the perturbative regime of small contrast (that is when a is a perturbation of the identity matrix). Finally, I will present some theoretical applications (optimal growth of the correctors, quantitative two-scale expansion) as well as numerical applications in stochastic homogenization.

- [1] S. Armstrong, T. Kuusi, J.-C. Mourrat. *Quantitative stochastic homogenization and large-scale regularity*, vol. 352. Springer, 2019.
- [2] N. Clozeau. *Optimal decay of the parabolic semigroup in stochastic homogenization for correlated coefficient fields*. arXiv preprint arXiv :2102.07452, 2021.
- [3] A. Gloria, F. Otto. *The corrector in stochastic homogenization : optimal rates, stochastic integrability, and fluctuations*. arXiv preprint arXiv :1510.08290, 2015.