Boundary stabilization of a one-dimensional cross-diffusion system in a moving domain using backstepping

Jean CAUVIN-VILA, CERMICS, Ecole des Ponts ; Inria MATHERIALS - Paris

We are interested in a particular one-dimensional cross-diffusion system that was proposed and studied in [1] to model the Physical Vapor Deposition (PVD) process used e.g for the fabrication of thin film crystalline solar cells. The procedure works as follows : a wafer is introduced in a hot chamber where several chemical elements are injected under a gaseous form. As the latter deposit on the substrate, a heterogeneous solid layer grows upon it. In the model, the solid layer is composed of $n+1$ different chemical species and occupies a domain of the form $(0, e(t)) \subset \mathbb{R}_+$, where $e(t) > 0$ denotes the thickness of the film and is determined by the fluxes ϕ_i of atoms that are absorbed at the surface of the layer :

$$e(t) = e_0 + \int_0^t \sum_{i=0}^n \phi_i(s) ds,$$

The cross-diffusion equation in the bulk, together with the flux boundary conditions, form the system :

$$\begin{cases} \partial_t u - \partial_x (A(u)\partial_x u) = 0, & \text{for } t \in \mathbb{R}_+^*, \ x \in (0, e(t)), \\ (A(u)\partial_x u)(t, 0) = 0, & \text{for } t \in \mathbb{R}_+^*, \\ (A(u)\partial_x u)(t, e(t)) + e'(t) u(t, e(t)) = \phi(t), & \text{for } t \in \mathbb{R}_+^*. \end{cases} \tag{1}$$

In [1], global existence of weak solutions was proved, adapting the *boundedness-by-entropy method*([4]). The authors also proved long-time asymptotics in the case of constant external fluxes : the solution converges (in a *rescaled* L^1 sense) to constant stationnary states with rate at least $\mathcal{O}(\frac{1}{\sqrt{t}})$.

In this poster, we are interested in achieving better rates of convergence (e.g exponential or finite time) to stationnary states by controlling the external gas fluxes ϕ_i. We will present the results obtained for the linearized system, following the *backstepping* approach of [3]. This is a joint work with Virginie Ehrlacher and Amaury Hayat at Ecole des Ponts.

Ongoing work is concerned with the treatment of the nonlinear terms in (1). In the future, we would like to propose and study a higher dimensional model for PVD that would include *surfacing diffusion effects*([2]).

[1] A. Bakhta, V. Ehrlacher. *Cross-diffusion systems with non-zero flux and moving boundary conditions.* ESAIM : Mathematical Modelling and Numerical Analysis, **52(4)**, 1385–1415, 2018.

[2] J. Cauvin-Vila. *Cross-diffusion systems in moving domains.* Ph.D thesis, Ecole des Ponts et Chaussees, Inria, In preparation.

[3] J.-M. Coron, H.-M. Nguyen. *Null controllability and finite time stabilization for the heat equations with variable coefficients in space in one dimension via backstepping approach.* Archive for Rational Mechanics and Analysis, **225(3)**, 993–1023, 2017.

[4] A. Jüngel. *The boundedness-by-entropy method for cross-diffusion systems.* Nonlinearity, **28(6)**, 1963, 2015.

Contact : jean.cauvin-vila@enpc.fr