Regularity for optimal compliance problems with length penalization

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In this talk, I will present some new results concerning the regularity and topological structure of minimizers of the optimal p-compliance problem with length penalization. In 2017, Antonin Chambolle, Jimmy Lamboley, Antoine Lemenant and Eugene Stepanov proved that in dimension 2 for $p = 2$, locally inside a given domain a minimizer, containing at least two points, of the optimal 2-compliance problem with length penalization is a finite union of $C^{1,\alpha}$ curves that can only meet at their ends, by sets of three and with 120° angles (see [4]). Recently, in [3] with Antoine Lemenant we have partially generalized this result in dimension 2 for all exponents $p \in (1, +\infty)$, namely it was proved that in dimension 2 every solution of the optimal p-compliance problem with length penalization cannot contain closed loops, is Ahlfors regular if contains at least two points (up to the boundary for a Lipschitz domain), and it is $C^{1,\alpha}$ regular at $\mathcal{H}^1$-a.e. point inside a given domain for every $p \in (1, +\infty)$. This result was partially generalized by the speaker in [2]. It was proved that in any spatial dimension $N \geq 2$ every optimal set cannot contain closed loops, and it is $C^{1,\alpha}$ regular at $\mathcal{H}^1$-a.e. point inside a given domain for every $p \in (N-1, +\infty)$. Furthermore, the importance of the connectedness assumption in the statement of the optimal p-compliance problem in both its penalized and constrained forms for the existence of solutions in any spatial dimension $N \geq 2$, for every $p \in (N-1, +\infty)$ and for the sharp integrability assumption on the source term was proved in [1]. In this talk, I will try to give a brief overview of these results.

[1] B. Bulanyi. *On the importance of the connectedness assumption in the statement of the optimal p-compliance problem.* J. Math. Anal. Appl., to appear. arXiv :2101.04678, 2021.

[2] B. Bulanyi. *Partial regularity for the optimal p-compliance problem with length penalization*, Preprint, arXiv :210104231 2021.

[3] B. Bulanyi, A. Lemenant. *Regularity for the planar optimal p-compliance problem.* ESAIM COCV, to appear. arXiv :1911.09240, 2021.

[4] A. Chambolle, J. Lamboley, A. Lemenant, E. Stepanov. *Regularity for the optimal compliance problem with length penalization.* SIAM J. Math. Anal., **49(2)**, 1166–1224, 2017.

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