Weak and Strong Solution for a Magnetohydrodynamic Problem

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We study a model of magnetohydrodynamics problem. One of the advantages of this type of model is that it is used in many applications and plays a crucial role in astrophysics, geophysics, planetary magnetism, engineering and controlled nuclear fusion. It describes the behavior of a fluid conducting electric current in the presence of electromagnetic fields. More precisely, we prove the existence of solution to the problem

$$\begin{cases} -\nu\Delta\mathbf{u} + (\mathbf{u}\cdot\nabla)\,\mathbf{u} - \dfrac{1}{\rho\mu}(\mathbf{B}\cdot\nabla)\,\mathbf{B} + \dfrac{1}{2\rho\mu}\nabla\left(|\mathbf{B}|^2\right) + \dfrac{1}{\rho}\nabla\pi = \mathbf{f} & \text{in } \Omega, \\ -\lambda\Delta\mathbf{B} + (\mathbf{u}\cdot\nabla)\,\mathbf{B} - (\mathbf{B}\cdot\nabla)\,\mathbf{u} = \mathbf{k} & \text{in } \Omega, \\ \text{div } \mathbf{u} = 0, \quad \text{div } \mathbf{B} = 0 & \text{in } \Omega, \end{cases}$$

with Dirichlet boundary condition for the velocity and Navier type conditions for the magnetic field :

$$\mathbf{u} = \mathbf{0}, \quad \mathbf{B}\cdot\mathbf{n} = 0, \quad \mathbf{curl}\,\mathbf{B}\times\mathbf{n} = \mathbf{0} \quad \text{on } \Gamma,$$

The domain Ω is a bounded open set of $\mathbb{R}^3$ of class $C^{1,1}$ and possibly non simply-connected. For the mathematical analysis of the problem, we will moreover assume that the magnetic field satisfies the flow conditions through Σ_j :

$$\int_{\Sigma_j} \mathbf{B}\cdot\mathbf{n} = 0, \qquad 1 \leqslant j \leqslant J.$$

Where Σ_j, $1 \leqslant j \leqslant J$, denote some cuts such that the open set $\Omega^\circ = \Omega \setminus \bigcup_{j=1}^{J} \Sigma_j$ is simply-connected.
Our approach is to use the Leray-Schauder fixed point theorem. To obtain the compactness properties of the operator, one main tool is given by some estimates for weak vector potentials corresponding to vector fields belonging to some negative Sobolev spaces.
We also investigate the regularity of the solution in L^p-theory. More precisely, we will prove the existence of generalized solution in $\mathbf{W}^{1,p}(\Omega)$, for $p \geq 2$ and strong solution in $\mathbf{W}^{2,p}(\Omega)$ for $p \geq \dfrac{6}{5}$.

Références

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