Intermittency in Lagrangian stochastic models for turbulent flows: genuine characterization and design of a versatile numerical approach

Ludovic GOUDENÈGE, Fédération Maths. CentraleSupélec, CNRS - Gif-sur-Yvette
Roxane LETOURNEL, Fédération Maths. CentraleSupélec, CNRS - Gif-sur-Yvette
Marc MASSOT, CMAP, CNRS, Ecole Polytechnique, IP Paris - Palaiseau
Alexandre RICHARD, Fédération Maths. CentraleSupélec, CNRS - Gif-sur-Yvette
Aymeric VIE, EM2C, CentraleSupélec, CNRS - Gif-sur-Yvette
Remi ZAMANSKY, IMFT, CNRS - INPT - UPS - Toulouse

The characterization of intermittency in turbulence has its roots in the K62 theory, and if no proper definition is to be found in the literature, statistical properties of intermittency were studied and models were developed in attempt to reproduce it. The first contribution of this work is to propose a requirement list to be satisfied by models designed within the Lagrangian framework.
Multifractal stochastic processes [5, 4] are a natural choice to retrieve multifractal properties of the dissipation. Among them, following the proposition of [3], we investigate the Gaussian Multiplicative Chaos formalism, which requires the construction of a log-correlated stochastic process X_t. The fractional Gaussian noise of Hurst parameter $H = 0$ is of great interest because it leads to a log-correlation for the logarithm of the process. Inspired by the approximation of fractional Brownian motion by an infinite weighted sum of correlated Ornstein-Uhlenbeck processes, our second contribution is to propose a new stochastic model : $X_t = \int_0^\infty Y_t^x k(x)dx$, where Y_t^x is an Ornstein-Uhlenbeck process with speed of mean reversion x and k is a kernel. A regularization of $k(x)$ is required to ensure stationarity, finite variance and logarithmic auto-correlation. A variety of regularizations are conceivable, and we show that they lead to the aforementioned multifractal models.
To simulate the process, we eventually design a new approach relying on a limited number of modes for approximating the integral through a quadrature $X_t^N = \sum_{i=1}^N \omega_i Y_t^{x_i}$, using a conventional quadrature method. This method can retrieve the expected behavior with only one mode per decade, making this strategy versatile and computationally attractive for simulating such processes, while remaining within the proposed framework for a proper description of intermittency.
An article on this topic has been submitted and is under review [2], the preprint is available on HAL and Arxiv. A companion paper is also in preparation [1], where we propose a synthesis of the mathematical key results and their justification in terms of stochastic calculus.

[1] L. Goudenège, R. Letournel, A. Richard. *Intermittency in a stochastic modelling of turbulence.* In preparation, 2021.

[2] R. Letournel, L. Goudenège, R. Zamansky, A. Vié, M. Massot. *Revisiting the framework for intermittency in Lagrangian stochastic models for turbulent flows : a way to an original and versatile numerical approach.* Submitted in Physical Review E. arXiv :2103.15562, 2021.

[3] B. B. Mandelbrot, J. W. Van Ness. *Fractional Brownian motions, fractional noises and applications.* SIAM review, **10(4)**, 422–437, 1968. doi :10.1137/1010093.

[4] R. M. Pereira, L. Moriconi, L. Chevillard. *A multifractal model for the velocity gradient dynamics in turbulent flows.* Journal of Fluid Mechanics, **839**, 430–467, 2018. doi :10.1017/jfm.2018.12.

[5] F. G. Schmitt. *A causal multifractal stochastic equation and its statistical properties.* European Physical Journal B, **34(1)**, 85–98, 2003. doi :10.1140/epjb/e2003-00199-x.

Contact : goudenege@math.cnrs.fr