Discrete and Potential mean field games

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Framework : In this work we study the class of discrete and potential mean field games. By discrete we mean discrete time and finite state space. The class of mean field games problems was introduced by J-M. Lasry and P-L. Lions in [4] and M. Huang, R. Malhamé, and P. Caines to study interactions among a large population of players. Our contribution is motivated by the study of a discrete model in itself but also by numerical aspects related to the discretization of mean field games problems. In contrast discrete time and finite state space mean field games have been poorly investigated. In a seminal work, D. Gomes, J. Mohr and R. R. Souza [2] study the existence and convergence to a Nash equilibrium via a fixed point approach. Economic models "à la Cournot", considering interactions between the agents via a price variable, have recently received particular attention [1]. Potential or variational mean field games is a class of games whose mean field game system can be interpreted as a first order condition of a control problem. The first formulation of this variational problem was given in [5]. It has been widely studied in the case of interactions through the state but up to our knowledge there are only two references available concerning interactions through the control [1, 3]. In this setting propositions have been made to study models with density constraints (also called hard constraints).

Main contributions : We propose a potential and discrete framework which was not consider before. There are very few results about discrete mean field games and up to our knowledge it is the first contribution about potential and discrete mean field games considering local coupling through the state or non-local coupling through the controls. In addition we take into account domain constraints on the congestion and price potentials. Up to our knowledge it is also the first contribution about hard constraints on prices which is of particular interest for the electricity market for example. Indeed in those markets the demand has to stay under a certain level otherwise black-out appears. We are able to show that from any primal and dual solutions to the primal and dual problem one can construct a solution to the mean field game system. Finally the last contribution of this work is to present numerical results. We adapt ADMM and ADM-G to our framework. We also study two Chambolle-Pock algorithms : we consider an euclidean setting and an entropic setting. Two problems are under study. A first mean field game problem with local coupling through the state and a second mean field game with non-local coupling through the control ("Cournot" mean field game in our case). Up to our knowledge this contribution is the first one about potential "Cournot" mean field games.

[1] J. F. Bonnans, S. Hadikhanloo, L. Pfeiffer. *Schauder estimates for a class of potential mean field games of controls.* Applied Mathematics & Optimization, pp. 1–34, 2019.

[2] D. A. Gomes, J. Mohr, R. R. Souza. *Discrete time, finite state space mean field games.* Journal de mathématiques pures et appliquées, **93(3)**, 308–328, 2010.

[3] P. J. Graber, C. Mouzouni. *Variational mean field games for market competition.* In *PDE models for multi-agent phenomena*, pp. 93–114. Springer, 2018.

[4] J.-M. Lasry, P.-L. Lions. *Jeux à champ moyen. i–le cas stationnaire.* Comptes Rendus Mathématique, **343(9)**, 619–625, 2006.

[5] J.-M. Lasry, P.-L. Lions. *Jeux à champ moyen. ii–horizon fini et contrôle optimal.* Comptes Rendus Mathématique, **343(10)**, 679–684, 2006.

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