On the incompressible limit for a tumour growth model incorporating convective effects

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We present a porous medium model with applications to tissue movement and tumour growth, [2]. The model is based on the standard fluid mechanics approach to living tissues, [5, 6]. We extend the analysis proposed in 2014 by Perthame, Quirós, and Vázquez, [4], by incorporating the advective effects caused, for instance, by the presence of nutrients, oxygen, or a chemoattractant. Passing to the singular limit for a stiff pressure law (incompressible limit), it is possible to bridge a link between the density-based model and a free-boundary problem of Hele-Shaw type. Our result extends those of [1, 3] thanks to weaker assumptions and a more general setting. In particular, we are able to recover the so-called *complementarity relation*, which allows to derive the pressure through an elliptic equation. To this end, we prove the strong compactness of the pressure gradient, blending two different techniques : an extension of the usual Aronson-Bénilan estimate in an L^3-setting and an L^4-uniform bound of the pressure gradient.

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