# Existence and regularity of a magnetohydrodynamic with Navier boundary condition in 2-D

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Magnetohydrodynamic (MHD) is the discipline studying the behaviour of conductive fluids of electricity when their movement is coupled to the electromagnetic field. Here we study in $\Omega$, a multi-connected two dimensional domain, the existence of solutions for a MHD coupling an equation of polymer aqueous solution with Maxwell equation of electromagnetic. These equations are presented, in the stationary case, as the following :

$$\begin{aligned} & -\nu\Delta\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)(\boldsymbol{u}-\alpha\Delta\boldsymbol{u}) + \nabla\pi - (\boldsymbol{B}\cdot\nabla)\boldsymbol{B} + \frac{1}{2}\nabla(|\boldsymbol{B}|^2) = \boldsymbol{f} && \text{in } \Omega, \\ & -\Delta\boldsymbol{B} - (\boldsymbol{B}\cdot\nabla)\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{B} + \nabla\theta = 0 && \text{in } \Omega, \\ & \operatorname{div} \boldsymbol{u} = 0, \quad \operatorname{div} \boldsymbol{B} = 0 && \text{in } \Omega, \end{aligned}$$

where $\boldsymbol{u}$ and $\boldsymbol{B}$ are the velocity field and the magnetic field, $\pi$ is the pressure of the fluid, $\theta$ is an unknown function related to the motion of heavy ions and $\boldsymbol{f}$ is the external force acting on the fluid. We study the existence of solutions $(\boldsymbol{u}, \boldsymbol{B}, \pi, \theta)$ in $\boldsymbol{H}^2(\Omega)\times\boldsymbol{H}^2(\Omega)\times\boldsymbol{L}^2(\Omega)\times\boldsymbol{H}^1(\Omega)$ with the Navier-type boundary conditions :

$$\begin{aligned} \boldsymbol{u}\cdot\boldsymbol{n} = 0, \qquad \operatorname{curl} \boldsymbol{u} = 0, \qquad \text{on } \partial\Omega \\ \boldsymbol{B}\cdot\boldsymbol{n} = 0, \qquad \operatorname{curl} \boldsymbol{B} = 0, \qquad \text{on } \partial\Omega. \end{aligned}$$

To solve our problem we need some estimations related to the Stokes associated problem. One of the difficulties is the geometry of the domain, supposed here non simply connected. On the other hand, it is shown an additional regularity in $\mathbf{W}^{2,p}(\Omega)$ for the magnetic field.

## Références

[1] Amrouche, C., Seloula, N.H. $\boldsymbol{L}^p$ theory for vector potentials and Sobolev's inequality for vector fields : Application to the Stokes equations with pressure boundary conditions. Mathematical Models and Methods in Applied sciences, Vol. 23, N. 1 (2013) 37-92

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