A TREFFTZ METHOD WITH RECONSTRUCTION OF THE NORMAL DERIVATIVE APPLIED TO ELLIPTIC EQUATIONS

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There are many classical numerical methods for solving boundary value problems on general domains. The Trefftz method is an approximation method for solving linear boundary value problems arising in applied mathematics and engineering sciences. This method consists to approximate the exact solution through a linear combination of trial functions satisfying exactly the governing differential equation. One of the advantages of this method is that the number of trial functions per cell is $O(m)$, asymptotically much less than the quadratic estimate $O(m^2)$ for finite element and discontinuous Galerkin approximations. For a Laplace model equation, we present a high order Trefftz method with quadrature formula for calculation of normal derivative at interfaces. We introduce a discrete variational formulation and study the existence and uniqueness of the discrete solution. A priori error estimate is then established and finally, several numerical experiments are shown.

Theorem 1. *Let $p \in W^{m+1,\infty}(\Omega)$ the exact solution of the Laplace Problem and p_h be the discrete solution of the Trefftz Problem. Assume the Gauss-Legendre quadrature formula has $m_1 \geq \frac{8}{3}m$ points, one has the error estimate*

$$\|p - p_h\|_{DG} \leq Ch^{2m/3},$$

where C is a positive constant.

See the proof in [2], and we used these two papers [1],[3].

[1] C. Buet, B. Després, G. Morel. *Trefftz discontinuous galerkin method for friedrichs systems with linear relaxation : Application to the p1 model.* Computational Methods in Applied Mathematics, 2018.

[2] B. Després, M. E. Ghaoui, T. Sayah. *A trefftz method with penalisation of the jump of traces, in preparation.*

[3] R.Herbin. *Analyse numérique des équations aux dérivées partielles.* Engineering school. Marseille, 2011.

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