An SDP relaxation of an optimal power flow problem for distribution network

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In this work, we are interested in optimal power flow problems in an electricity distribution network. The optimization problem of interest consist in minimizing the total network line losses under some constraints such as power flow equations, which results from Kirchhoff law and appear as an equality constraint, and some other physical constraints such as limitations on voltage angles, voltages magnitudes and power injections.
Some of the constraints are non-convex and therefore make the problem also non-convex. This problem as defined can be difficult to solve, and even by figuring out a method to solve it, the solution obtained is not necessarily a global minimum.
There exists many approaches to convexify the original problem, one of them is the convex relaxation. It consists in replacing the set of non-convex constraints by a bigger convex set, and for which we can easily find an efficient algebraic representation. The convex relaxation is then said to be exact if the optimal value of the original problem is equal to the optimal value of the convexified problem.
To achieve this method, we reformulate the objective function and the constraints of the original problem, in terms of positive semi-definite matrix traces, to which we add a rank constraint. The convex relaxation will then consist in removing this rank constraint. This will lead to a positive semi-definite optimization problem that can be solved using efficient algorithms such as the interior point algorithm. This method is called an SDP relaxation and was proposed by [1].
We show that under a condition on the objectif function and conditions on the physics of the network, the SDP relaxation is exact. To this end, we explore the geometry of the feasible set of the problem via its Pareto front [2, 3]. The main result that we prove is that the feasible set of the original problem and the feasible set of its convexification share the same Pareto front. We conclude by presenting some numerical examples to highlight the efficiency of the proposed method.

[1] X. Bai, H. Wei, K. Fujisawa, Y. Wang. *Semidefinite programming for optimal power flow problems.* International Journal of Electrical Power & Energy Systems, **30(6-7)**, 383–392, 2008.

[2] J. Lavaei, D. Tse, B. Zhang. *Geometry of power flows in tree networks.* In *2012 IEEE Power and Energy Society General Meeting*, pp. 1–8. IEEE, 2012.

[3] J. Lavaei, D. Tse, B. Zhang. *Geometry of power flows and optimization in distribution networks.* IEEE Transactions on Power Systems, **29(2)**, 572–583, 2013.

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