Quantitative particle approximation of nonlinear Fokker-Planck equations with singular kernel

Christian OLIVERA, IMECC-UNICAMP - Campinas
Milica TOMASEVIC , CMAP, Ecole Polytechnique - Paris
Alexandre RICHARD, MICS, Ecole Centrale - Paris

In this talk, we are interested in the convergence of the empirical measure of moderately interacting particle systems with singular interaction kernels. First, we will present a result of quantitative convergence of the time marginals of the empirical measure of particle positions towards the solution of the limiting nonlinear Fokker-Planck equation. Second, we will discuss the well-posedness for the McKean-Vlasov SDE involving such singular kernels and the convergence of the empirical measure towards it (propagation of chaos).
These results only require very weak regularity on the interaction kernel, including the Biot-Savart kernel, and attractive kernels such as Riesz and Keller-Segel kernels in arbitrary dimension. For some of these important examples, this is the first time that a quantitative approximation of the PDE is obtained by means of a stochastic particle system which paves the way to numerical applications. In particular, this convergence still holds (locally in time) for PDEs exhibiting a blow-up in finite time.
After presenting the results, the talk will give a glimpse of techniques used in [1] to prove the above results that are based on a semigroup approach combined with a fine analysis of the regularity of infinite-dimensional stochastic convolution integrals.

[1] C. Olivera, A. Richard, M. Tomasevic. *Quantitative particle approximation of nonlinear fokker-planck equations with singular kernel.* Preprint ArXiv 2011.00537, 2020.

Contact : `milica.tomasevic@polytechnique.edu`