Propagation of moments for the magnetized Vlasov-Poisson system

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The Vlasov-Poisson system is a set of PDE's that govern the evolution of a cloud of particles in astrophysics or plasma physics. Firstly, by adapting the work of Lions, Perthame [2], we will detail how we proved propagation of velocity moments for the 3-dimensional Vlasov-Poisson system with an added uniform magnetic field $B = (0, 0, \omega)$ [5] .
The added magnetic field produces singularities at times which are the multiples of the cyclotron period $t = \frac{2\pi}{k}$. This is the main difficulty, and we get around it by noticing that our estimates depend only on the initial condition and constant parameters, which means our logarithmic estimate for the force field is true at all time. This result also allows to show propagation of regularity for the solution.
Secondly, in the non-constant case, we change our point of view and rely this time on a "Lagrangian" approach first introduced by Pfaffelmoser [4] to show the existence of classical solutions to Vlasov-Poisson, which translates to studying the characteristics in detail instead of looking at moments. More precisely, we use the optimal result of Pallard [3] to extend the propagation of velocity moments to the case of non-constant magnetic field.
Finally, we investigate the propagation of space moments first studied by Castella [1] to deal with solutions to Vlasov-Poisson with infinite kinetic energy.

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