Fast marching method for the 3D eikonal equation in elastic media with general anisotropy

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I present a numerical method for the eikonal equation in the context of seismic imaging. The solution to this eikonal equation corresponds to the first arrival time of a seismic wave for a propagation inside a medium with elastic properties defined by a Hooke tensor. This setting typically leads to metrics with anelliptical anisotropy, which has been a technical challenge for numerical solvers of the eikonal equation.
Fast Sweeping and Fast Marching algorithms are very efficient for solving the eikonal equation in an isotropic setting [6, 5]. Some numerical solvers using these algorithms have already been proposed for the eikonal equation with the anisotropy from geophysics, but require additional symmetries for the Hooke tensor [7, 2].
Our numerical solver proceeds in a single pass over the domain, similar to the Fast Marching method, and can tackle fully anisotropic Hooke tensors. It generalizes the solver for elliptic anisotropy from [3, 4]. We achieve quasi-linear computation time and third-order accuracy in smooth test cases [1].

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