Fully adaptive lattice Boltzmann methods with error control based on multiresolution analysis

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Lattice Boltzmann Methods (LBM) stand out for their simplicity, computational efficiency and ease of parallelization while offering the possibility of simulating complex phenomena. While they are optimal for uniform Cartesian meshes with a strong bond between spatial and temporal discretization, adapted meshes have traditionally been a stumbling block since it is difficult to predict the right physics through various levels of meshes. This fact complicates the crucial issue of reducing the computational cost and the memory impact by automatically coarsening the grid where a fine mesh is unnecessary, still ensuring the overall quality of the numerical solution through error control.
In this work [2, 1], we connect for the first time the field of lattice Boltzmann Methods to the adaptive multiresolution (MR) approach based on wavelets. To this end, we design a class of fully adaptive LBM methods with dynamic mesh adaptation and error control relying on multiresolution analysis. This wavelet-based approach allows to adapt the mesh based on the local regularity of the solution and leads to a very efficient compression of the solution without loosing its quality and with the preservation of the properties of the original LBM method on the finest grid. In particular, the collision phase is not affected due to its inherent local nature and because we do not modify the speed of the sound, contrarily to most of the LBM/Adaptive Mesh Refinement (AMR) strategies proposed in literature. Besides, an original use of the MR allows the scheme to resolve the proper physics by efficiently controlling the accuracy of the transport phase. This yields a general approach for a large spectrum of schemes and allows to propose a precise error analysis of the method, without the need for deep modifications on the reference scheme.
For the purpose of assessing the approach, we conduct a series of tests for various schemes and scalar and systems of conservation laws, both hyperbolic and parabolic, where solutions with shocks are to be found and local mesh adaptation is especially relevant. Theoretical estimates guaranteeing a precise control on the errors are retrieved while a reduced memory footprint is observed. The numerical strategy is implemented on a specific open-source platform called `SAMURAI`[1] with a dedicated data-structure relying on set algebra.

[1] T. Bellotti, L. Gouarin, B. Graille, M. Massot. *Multidimensional fully adaptive lattice boltzmann methods with error control based on multiresolution analysis.* arXiv preprint arXiv :2103.02903, 2021.

[2] T. Bellotti, L. Gouarin, B. Graille, M. Massot. *Multiresolution-based mesh adaptation and error control for lattice boltzmann methods with applications to hyperbolic conservation laws.* arXiv preprint arXiv :2102.12163, 2021.

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1. Examples and code available on `https://github.com/hpc-maths/samurai`