The discretized backstepping: the example of a system of two linear balance laws

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In this talk, we focus on the following system of two scalar transport equations coupled in the domain $\Omega = [0,1]$ and at the boundary :

$$\begin{cases} \partial_t R + \lambda_+ \partial_x R & = & M_{12} S \\ \partial_t S - \lambda_- \partial_x S & = & M_{21} R \\ R(t,0) & = & u(t) \\ S(t,1) & = & hR(t,1) \end{cases} \tag{1}$$

The problem is to find a control $u(t)$ to stabilize system (1) in finite time. More precisely, we aim at finding a control $u(t)$ such that :

$$\|R(t,\cdot)\|_{L^2([0,1])} + \|S(t,\cdot)\|_{L^2([0,1])} = 0, \ \forall t \geq \frac{1}{\lambda_+} + \frac{1}{\lambda_-}. \tag{2}$$

Using the backstepping method for PDEs, this question is solved in [1] where the authors find a control $u(t)$ and a bijective transformation to map system (1) to the following target system :

$$\begin{cases} \partial_t R + \lambda_+ \partial_x R & = & 0 \\ \partial_t S - \lambda_- \partial_x S & = & 0 \\ R(t,0) & = & 0 \\ S(t,1) & = & hR(t,1) \end{cases} \tag{3}$$

for which property (2) is obvious.
The main goal of this presentation is to apply the backstepping method presented before in a numerical context. More precisely, using the classical upwind scheme, system (1) is discretized and we find a discrete second order Volterra transform to map this system to a numerical version of (3). A result of approached finite time stabilization is given at the end of the presentation.

[1] R. Vazquez, M. Krstic, J. Coron. *Backstepping boundary stabilization and state estimation of a 2 x 2 linear hyperbolic system*. In *2011 50th IEEE Conference on Decision and Control and European Control Conference*, pp. 4937–4942, 2011. doi :10.1109/CDC.2011.6160338.

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