# Hyperbolic Quadrature Method of Moments for the one-dimensional kinetic equation

**Frédérique LAURENT**, EM2C & Fédé. de Math. - CNRS, CentraleSupélec - Gif-sur-Yvette
**Rodney O. FOX**, Iowa State University - USA

For applications such as rarefied gas dynamics or spray flows, there is an interest in developing moment methods from a kinetic description of the gas or the spray, since such methods can be very efficient. In this context, a solution is proposed here [1] to a longstanding open problem in kinetic theory, namely, given any set of realizable velocity moments up to order $2n$, a closure for the moment of order $2n+1$ is constructed for which the moment system found from the free-transport term in the one-dimensional (1-D) kinetic equation is globally hyperbolic and in conservative form. This is a reformulation and generalization of the Hyperbolic Quadrature Method of Moments (HyQMOM) introduced in a prior work [2].
The HyQMOM closure is defined based on the properties of the monic orthogonal polynomials $Q_n$ that are uniquely defined by the velocity moments up to order $2n-1$. Thus, HyQMOM is strictly a moment closure and does not rely on the reconstruction of a velocity distribution function with the same moments. Moreover, one can show the hyperbolicity of the corresponding system, at least for $n \leq 11$ and the good behavior of the eigenvalues of the problem, in particular when the moment vector tends to the boundary of the moment space, property that can be important for applications such as sprays. An efficient algorithm for computing the moment of order $2n+1$ from the moments up to order $2n$ is developed, based on the Chebyshev algorithm. The analytical solution to a 1-D Riemann problem is used to show convergence of the HyQMOM closure with increasing $n$.

[1] R. O. Fox, F. Laurent. *Hyperbolic quadrature method of moments for the one-dimensional kinetic equation*, 2021. Working paper or preprint.

[2] R. O. Fox, F. Laurent, A. Vié. *Conditional hyperbolic quadrature method of moments for kinetic equations*. Journal of Computational Physics, **365**, 269–293, 2018.

Contact : `frederique.laurent@centralesupelec.fr`