Extension of a Roe type scheme with low Mach correction to the HRM two-phase flow model

Thomas GALIÉ, CEA/DES/ISAS/DM2S/STMF/LMEC - Saclay

This work deals with the numerical approximation of the Homogeneous Relaxation Model (HRM) which is a mixture model widely used for simulating two-phase flows. This four-equations model is derived from the six-equation two-fluid model and can be written under the following contracted form :

$$\partial_t U + \nabla \cdot F(U) = S(U), \tag{1}$$

where $U = (\rho, \rho c, \rho \mathbf{u}, \rho E)^T$ is the vector of the unknowns which are respectively the density of the mixture, the vapor mass, the momentum and the total energy $\rho E = \rho e + p$ of the mixture. The internal energy e and the pressure p are linked to the other thermodynamical variables thanks to the equation of state given under the form $f(\rho, \rho c, \rho e, p) = 0$. The convective fluxes F and source terms S are written as

$$F(U) = \begin{pmatrix} \rho \mathbf{u}^T \\ \rho c \mathbf{u}^T \\ \rho \mathbf{u} \otimes \mathbf{u} + pI \\ (\rho E + p)\, \mathbf{u}^T \end{pmatrix}, \quad S(U) = \begin{pmatrix} 0 \\ \Gamma \\ \rho \mathbf{g} + \tau_s \\ Q \end{pmatrix}.$$

Here we have noted Γ the source term of mass exchange between the two phases liquid and vapor, $\mathbf{g}$ the gravity vector, τ_s a source term for singular pressure loss in the domain and Q the energy source term. In order to numerically solve system (1), we use a classical finite volume approach as described in [4] and the Roe solver [3] for the approximation of the convective fluxes on each face of the mesh. As explained in [2], this type of scheme suffers from an accuracy problem in the low Mach number limit on 2D and 3D cartesian grids. A pressure correction applied to the numerical momentum flux enables to recover the right order of accuracy. However, by removing some spatial diffusion, checkerboard modes can appear and also be amplified by discontinuous source terms (as τ_s for instance). In order to get rid of these checkerboard modes, we want to extend the new corrected Roe scheme detailed in [1] to the resolution of the two-phase flow HRM system (1).

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Contact : `thomas.galie@cea.fr`