MAP5-UMR 8145
Mathématiques Appliquées à Paris 5
LABORATOIRE MAP5

Partial Matching in the Space of Varifolds

SMAI 2021, June 24th

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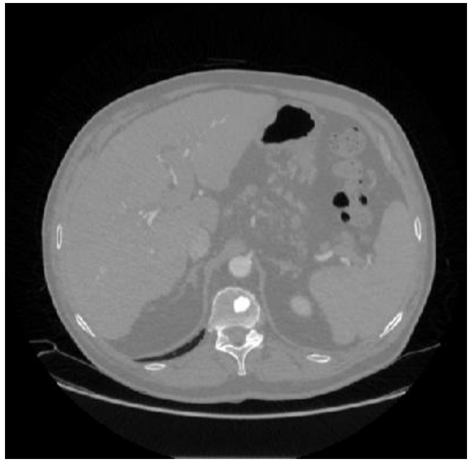
Thomas Benseghir and Vincent Jugnon (GE Healthcare)

Context

Partial Matching in Medical Images

CBCT

CT



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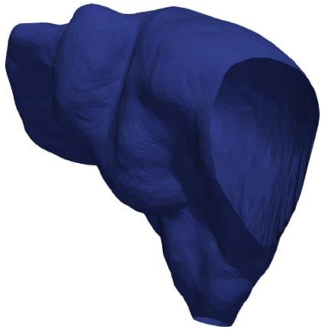

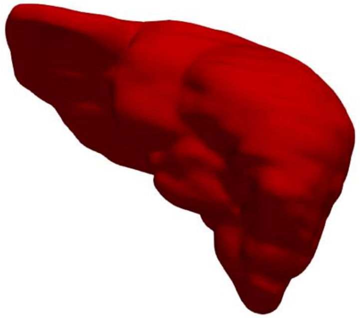

Segmented liver

Livers Surfaces

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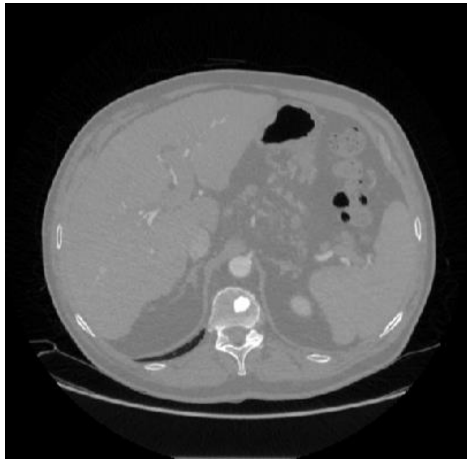

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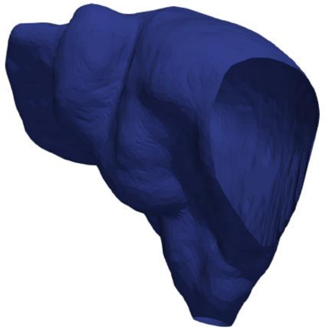

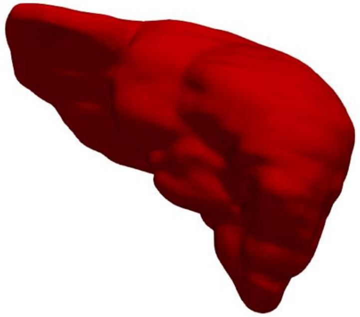

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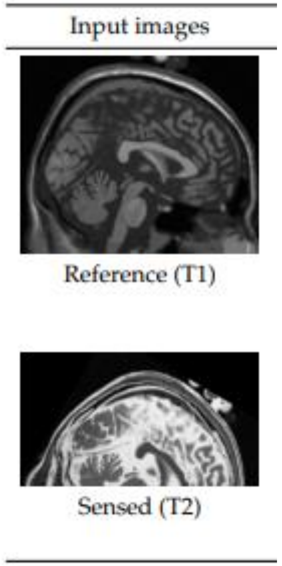

[Bashiri 2019]

Cerebral MRI

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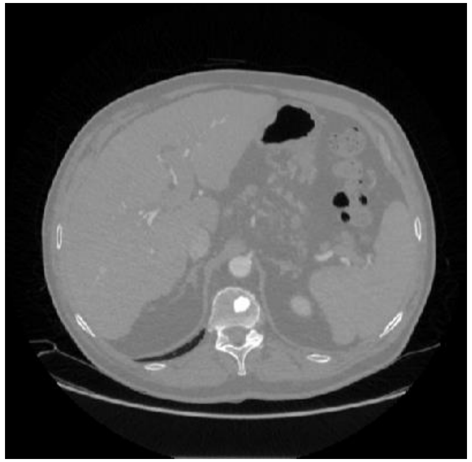

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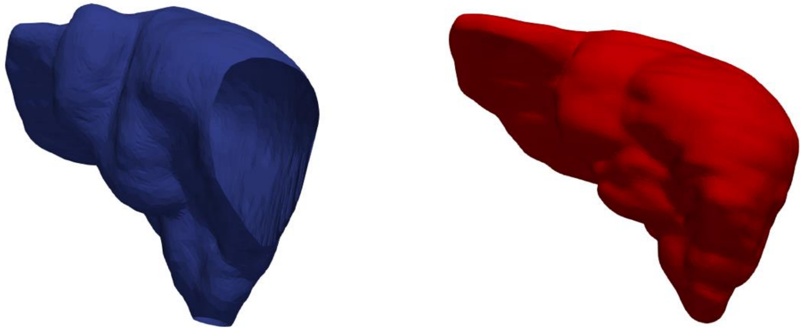

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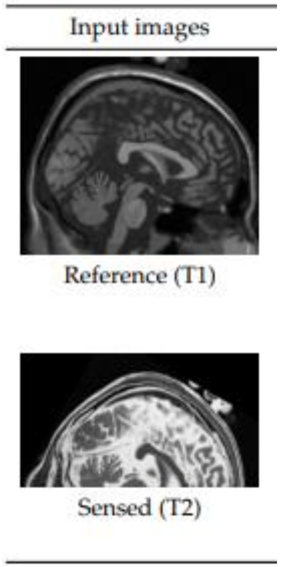

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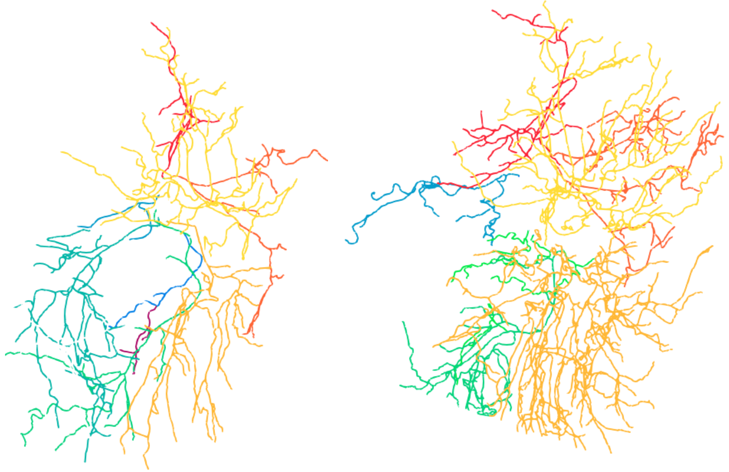

Pelvic Arterial Trees

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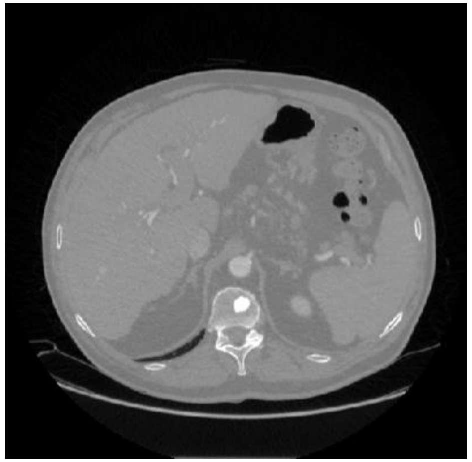

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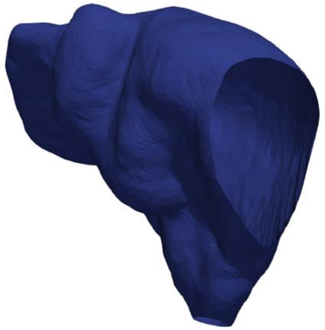

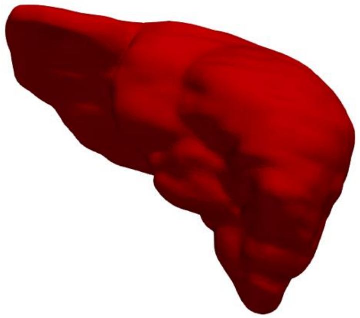

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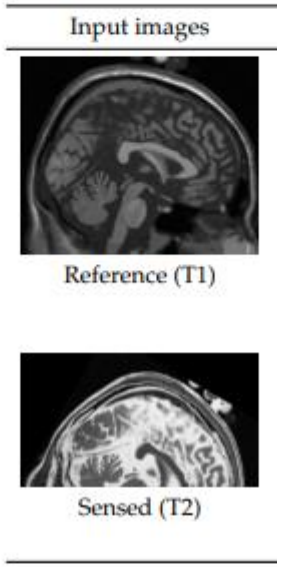

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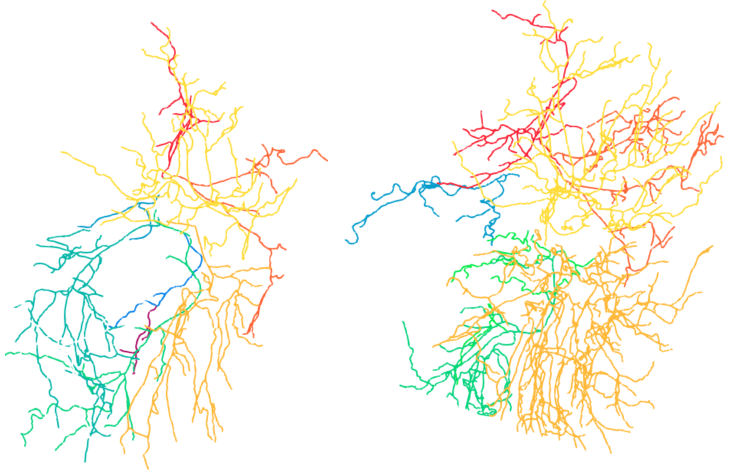

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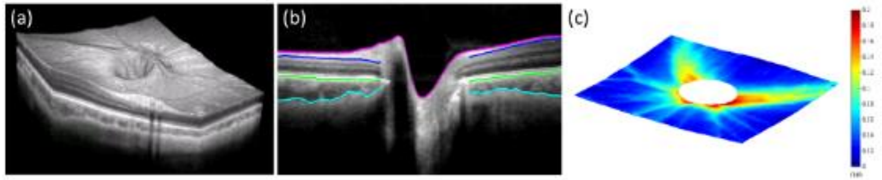


Retinal Nerve Fiber Layer [Lee 2017]

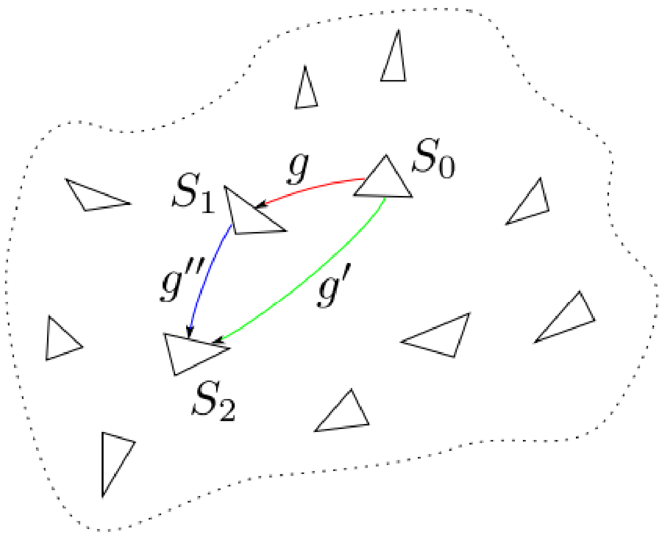
Outline

- The Shape Space
  - Large Diffeomorphic Deformation Metric Mapping
- Data fidelity metrics
  - Overview
  - Building Metrics to Compare Shapes
- Partial Matching
  - First Idea
  - Localized Partial Matching
  - Localized and Normalized Partial Matching
  - Applications

The Shape Space

The Large Deformations Diffeomorphic Metric Mapping (LDDMM)

See the shapes through the deformations between them [Grenander, Miller, Trouvé, Younes, Beg, …, Arguillère]



Distance between shapes through actions of deformations :

$$d_S(S_0, S_2) = inf\{d_G(id, g') \mid g'.S_0 = S_2\}$$
$$d_S(S_0, S_2) = E(S_0, S_2)$$

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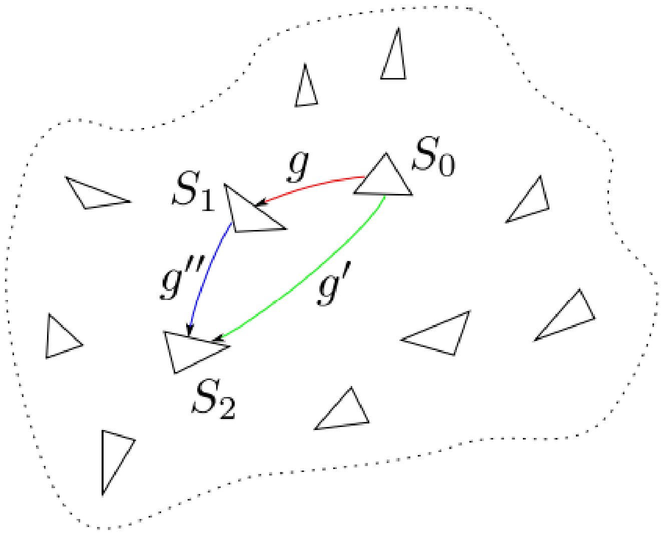

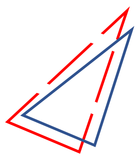

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Data attachment term: Being robust to noise, non-diffeomorphic changes…

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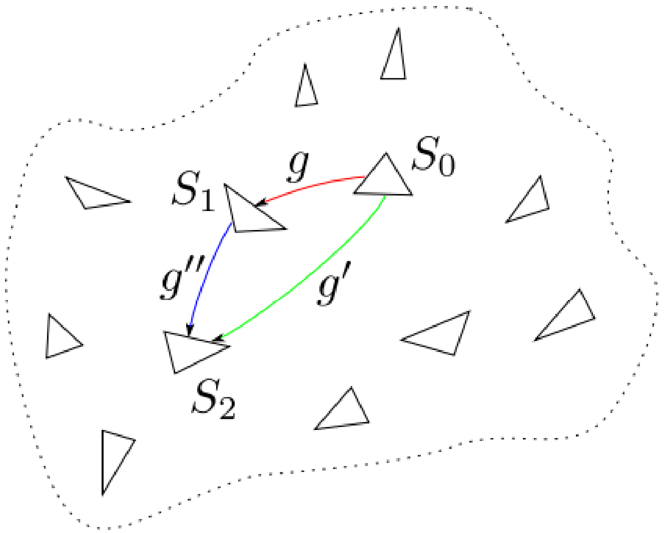

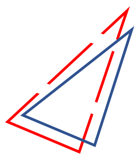

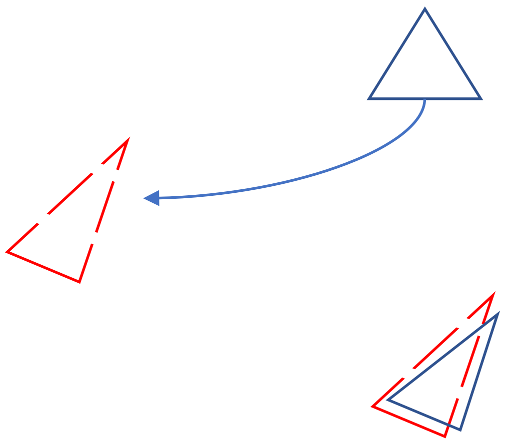

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Data attachment term: Being robust to noise, non-diffeomorphic changes…

Inexact Registration Framework: Minimize the energy

$E(S_0, S_2) + A(g'(S_0), S_2)$

Data fidelity metrics

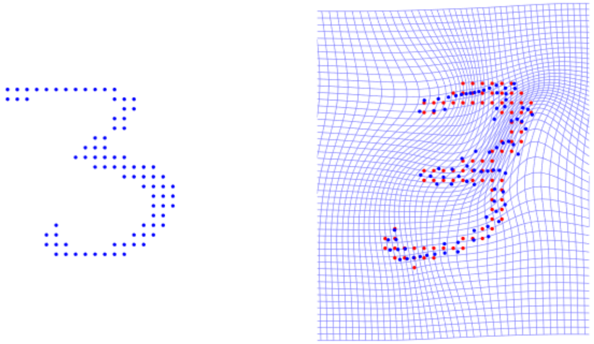
How to compare shapes ?

What we expect :

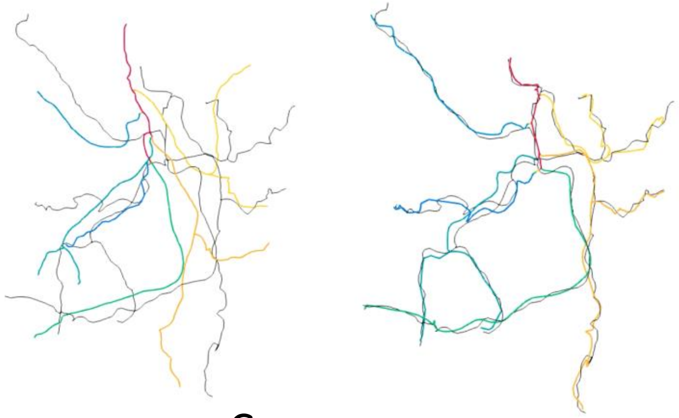
- Adapted to large type of data (landmarks, images, curves or surfaces)
- Independent to parametrization
- No need to compute points correspondences

Currents [Glaunès 2005]
Varifolds [Charon 2013]
Oriented Varifolds [Kaltenmark 2017]

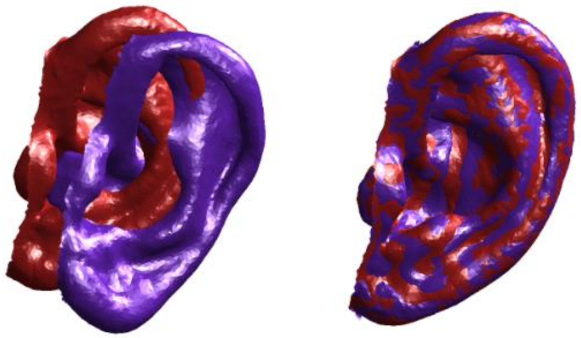
Some other metrics :
Normal Cycles [Roussillon 2017]
Optimal Transport as Data Term [Feydy 2017]
Square Root Velocity Functions [Srivastava 2012]



Landmarks



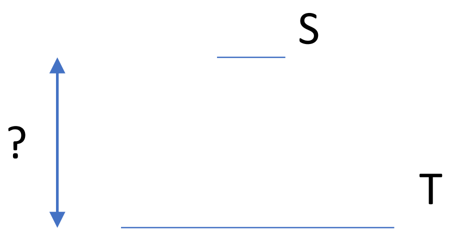
Curves



Surfaces

Data fidelity metrics

How can two shapes "see" each others ?



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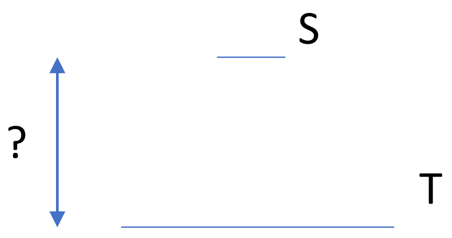

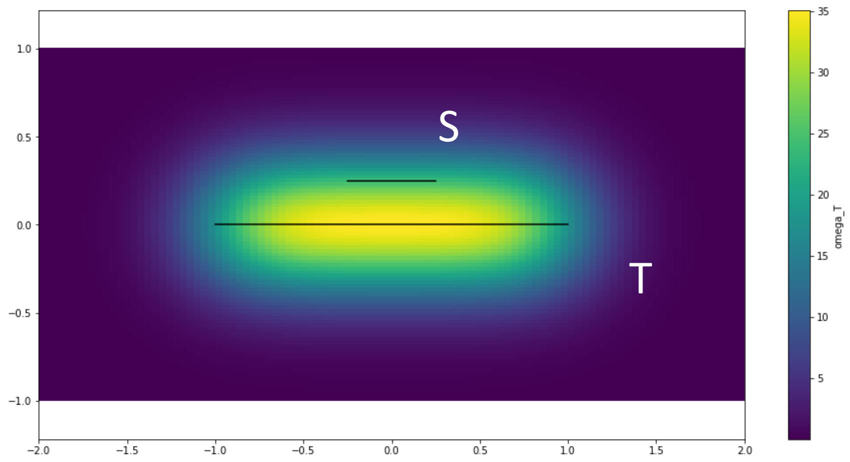

$$\omega_T(y) = \int_T e^{-\frac{\|x-y\|^2}{s^2}} dx$$

$$s = 0.4$$

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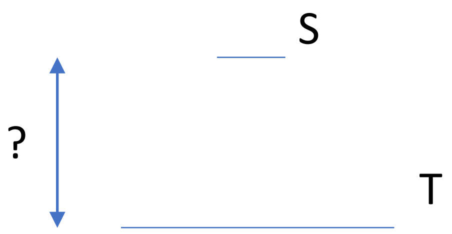

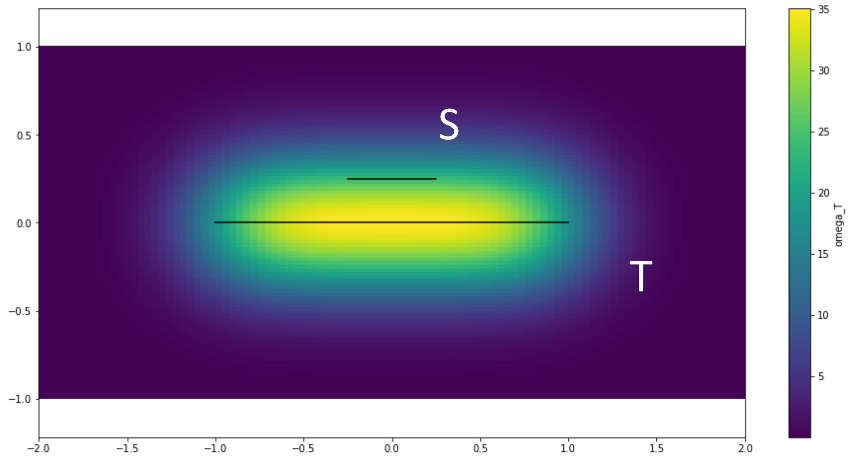

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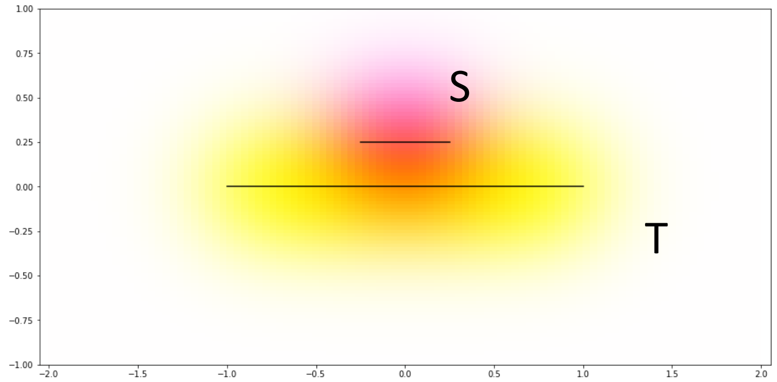

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$$\omega_S(y) = \int_S e^{-\frac{\|x-y\|^2}{s^2}} dx$$

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Data fidelity metrics

Riesz representation theorem COMBO **Aronzajn Theorem**

Reproducing Kernel Hilbert Spaces (RKHS)

Definition

Let $(H, \|\,.\,\|_H)$ be a Hilbert space of functions from a set X onto an euclidean space E of finite dimension.
H is a RKHS if $\forall(x, \alpha) \in X \times E$ the evaluation functions $\delta_x^\alpha : \omega \mapsto \omega(x)\alpha$ are continuous linear forms on H.

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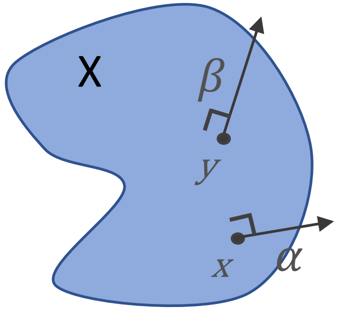
- To a RKHS $\mathbf{H}$ corresponds a unique RK $\boldsymbol{k_H}$
- $\forall(x,y) \in X \times X, \forall(\alpha,\beta) \in E \times E$: $\left\langle \boldsymbol{\delta_x^\alpha} \middle| \boldsymbol{\delta_y^\beta} \right\rangle_{\boldsymbol{H'}} = \boldsymbol{k_H}\big((\boldsymbol{x},\boldsymbol{\alpha}),(\boldsymbol{y},\boldsymbol{\beta})\big)$



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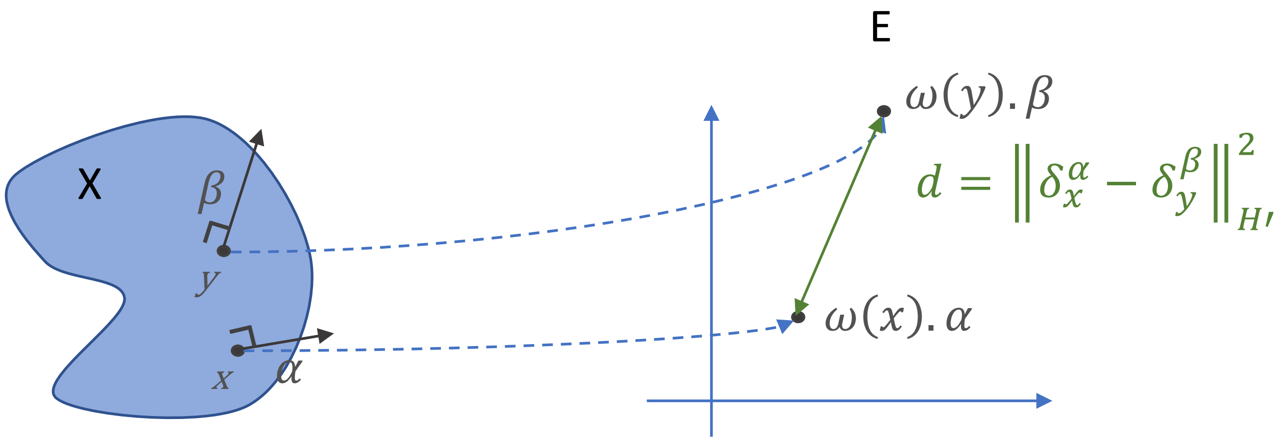

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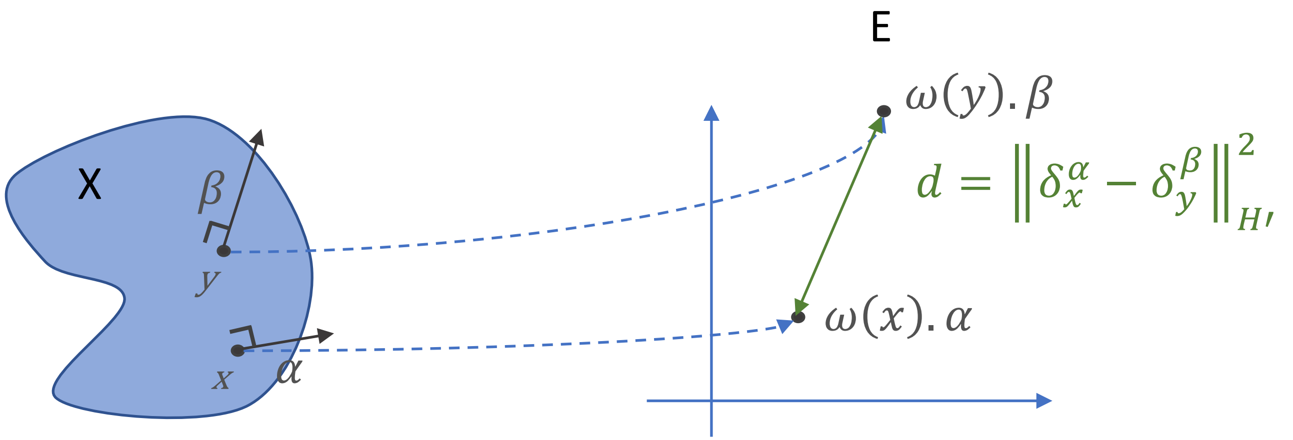
$\mu_X \in H'$,

$$\mu_X(\omega) = \int_X \boldsymbol{\delta_x^\alpha}(\omega)\, dvol^X(x)$$



Data fidelity metrics

N.B :
If k_H RK of H and $k_{\widehat{H}}$ RK of $\widehat{H}$, then $k_H \times k_{\widehat{H}}$ RK of $H \times \widehat{H} = W$

Building metrics to compare shapes

$$\omega \in W = C_0\left(\left(\mathbb{R}^d \times \mathbb{S}^{d-1}\right), \mathbb{R}\right), d = 2{,}3$$

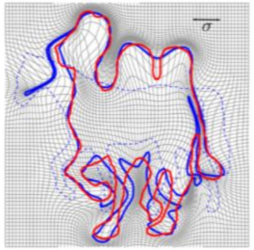
$$\|\mu_X\|^2_{W'} = \int_X \int_X k_W\big((x,\alpha),(y,\beta)\big) dvol^X(x) dvol^X(x)$$

$\forall (x,y) \in X \times X, \forall (\alpha,\beta) \in S^{\{d-1\}} \times S^{\{d-1\}}$:

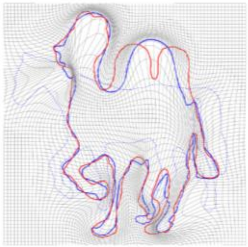
- $\langle k_W((x,\alpha),.) | k_W((y,\beta),.)\rangle_W = k_H(x,y).\langle \beta|\alpha\rangle^2$ Varifolds
- $\langle k_W((x,\alpha),.) | k_W((y,\beta),.)\rangle_W = k_H(x,y)\,.\,e^{-\frac{\langle \beta|\alpha\rangle^2}{\sigma^2}}$ Oriented Varifolds

In practice, X is a rectifiable subset of $\mathbb{R}^d$
And we are interested in kernel invariant to rotations and translations.

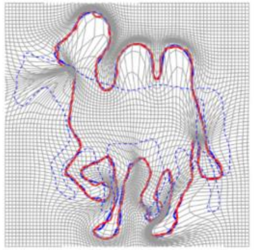
We take $k_H(x,y) = \tilde{k}_H\left(|x-y|_{\mathbb{R}^d}\right)$, for example $e^{-\frac{\|x-y\|^2}{s^2}}$.



Current

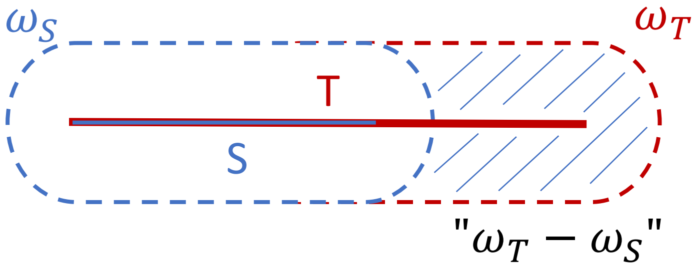


Unor varifold



Or varifold

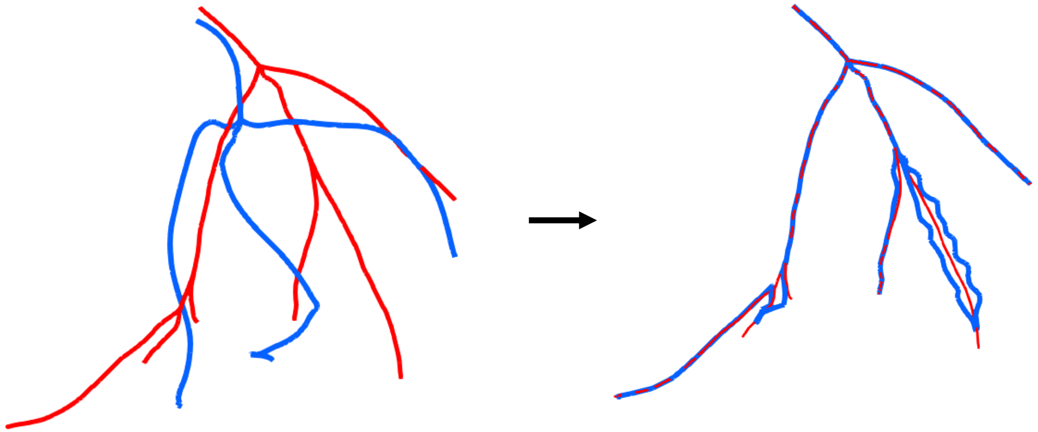
Partial Matching



Toward real world applications

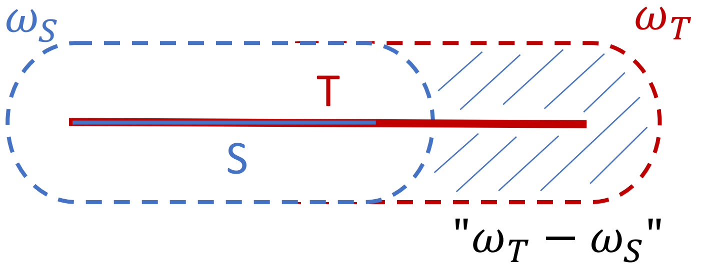
The data attachment term in the space of Varifolds :

$$A(S,T) = \|\mu_S - \mu_T\|^2_{W'} = \|\mu_S\|^2_{W'} - 2.\langle \mu_S | \mu_T \rangle + \|\mu_T\|^2_{W'}$$



Registration of a trimmed source (blue) onto a richer target (red).

Partial Matching



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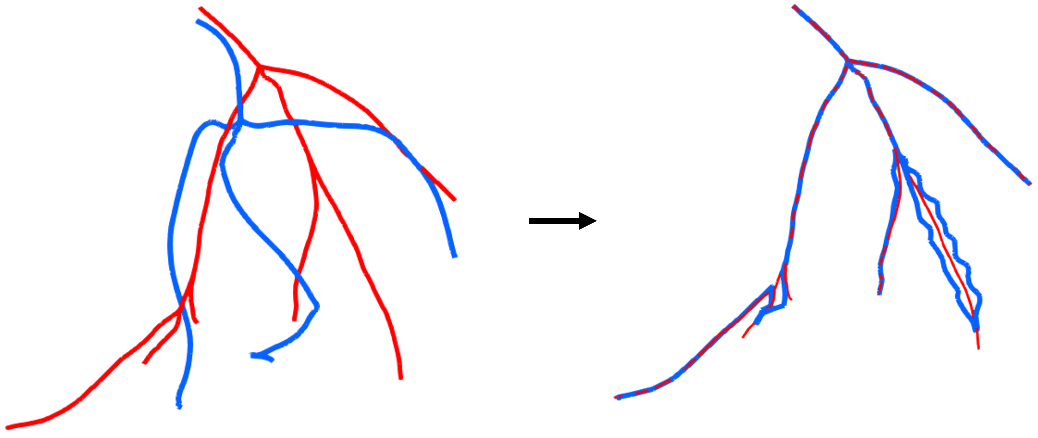

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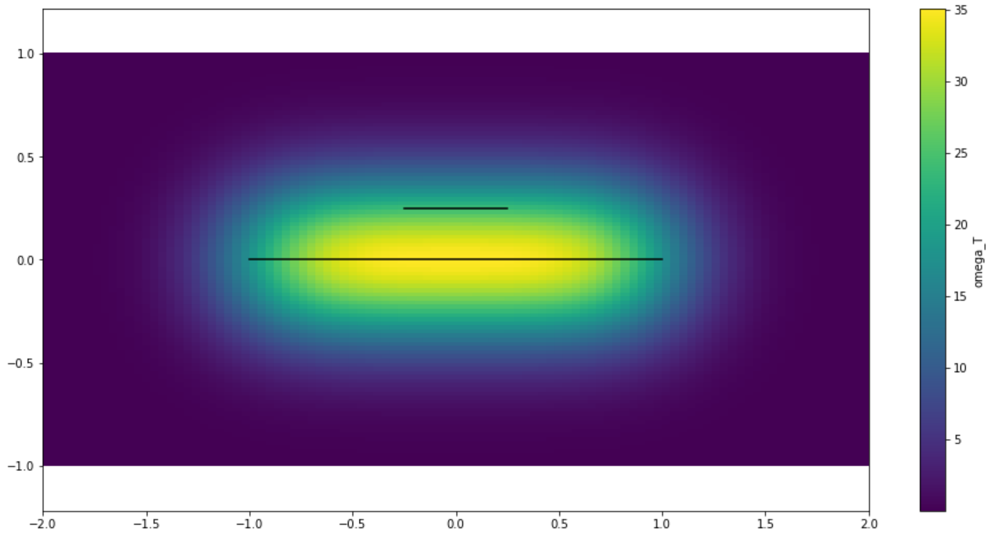

Partial matching dissimilarity function (first idea)

$$A(S,T) = (\ \|\mu_S\|^2_{W'} - \langle \mu_S | \mu_T \rangle\ )^2$$

$$= \langle \mu_S | \mu_S - \mu_T \rangle^2$$

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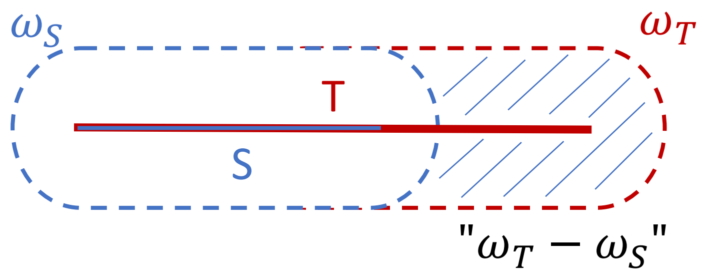


*Step by step : **1st idea***

The data attachment term in the LDDMM framework :

$$A(S,T) = \|\mu_S\|^2_{W'} - 2.\langle\mu_S|\mu_T\rangle + \|\mu_T\|^2_{W'}$$

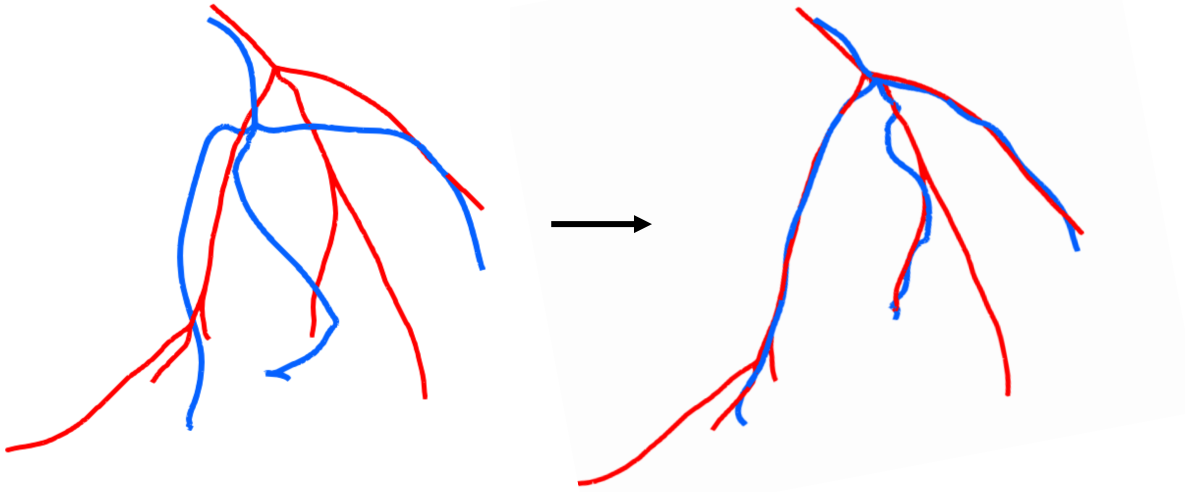
Partial matching dissimilarity function (first idea) :

$$A(S,T) = (\|\mu_S\|^2_{W'} - \langle\mu_S|\mu_T\rangle)^2$$

$$= \langle\mu_S|\mu_S - \mu_T\rangle^2$$

$$= \left(\int_S \omega_S(\boldsymbol{x}) - \omega_T(\boldsymbol{x})dx\right)^2$$

With $\boldsymbol{x} \in S \times \boldsymbol{T}_x S$



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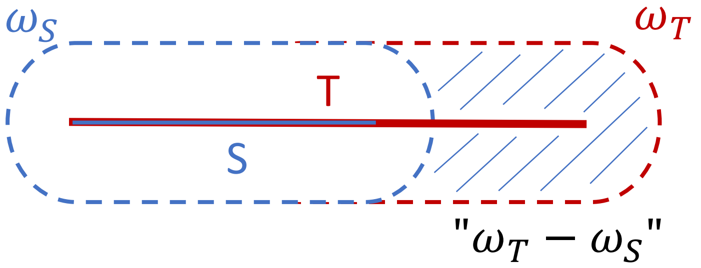

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Partial matching dissimilarity function (first idea) :

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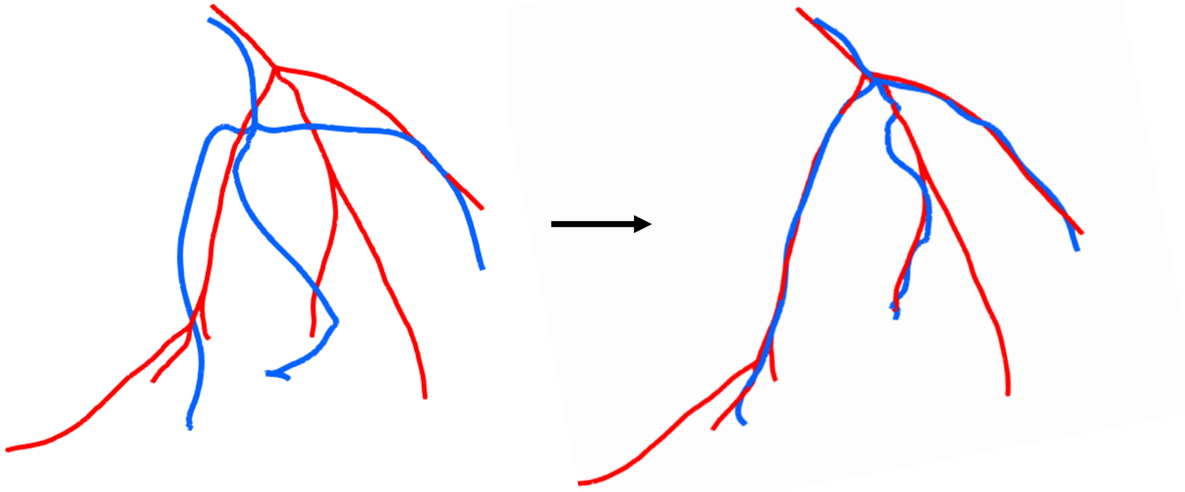
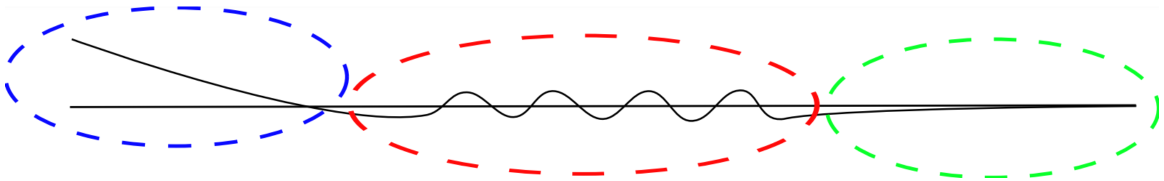
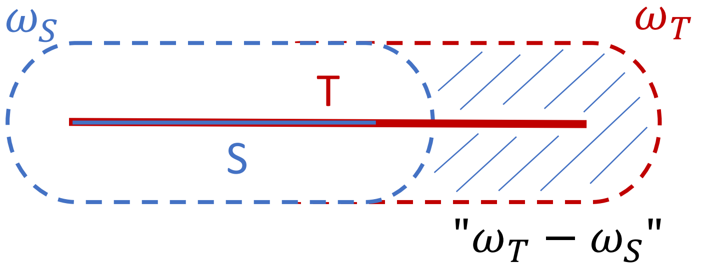


Registration of a trimmed source (blue) onto a richer target (red)



Partial Matching



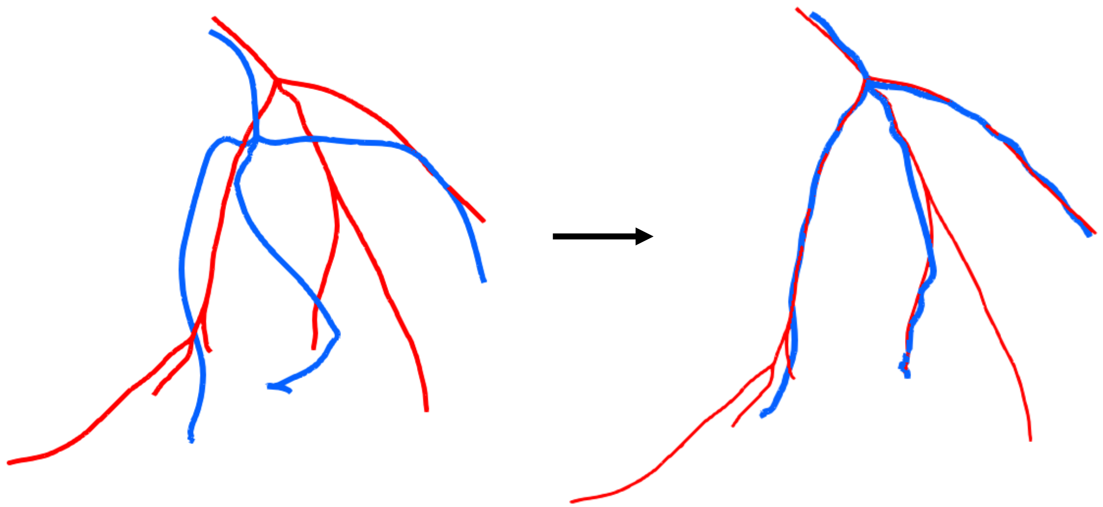
*Localized version : **definition***

Partial matching dissimilarity function (second idea):

$$\boldsymbol{A_1}(S,T) = \int_S g\big(\omega_S(\boldsymbol{x}) - \omega_T(\boldsymbol{x})\big)dx$$

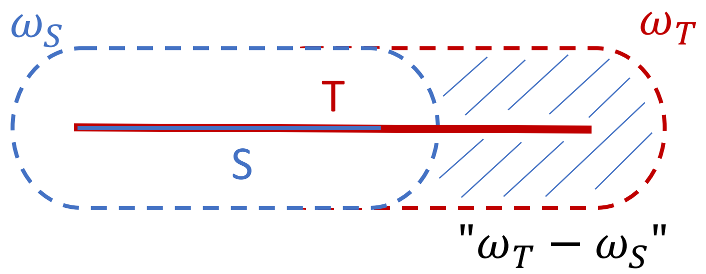
With $g : \mathbb{R} \to \mathbb{R}$

$$\mathrm{x} \mapsto \begin{cases} 0 \; if \; x \leq 0 \\ x^2 \; otherwise \end{cases}$$



Registration of a trimmed source (blue) onto a richer target (red)

Partial Matching



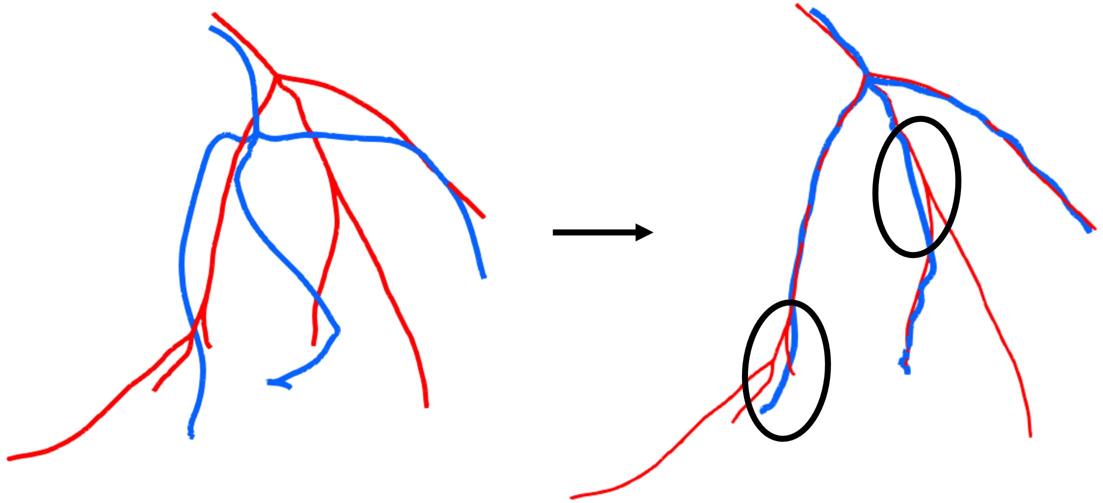
*Localized version : **the local mass problem***

Partial matching dissimilarity function (second idea):

$$A_1(S,T) = \int_S g(\omega_S(\boldsymbol{x}) - \omega_T(\boldsymbol{x}))dx$$

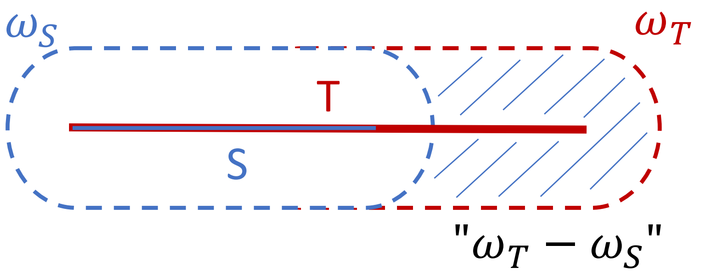
With $g : \mathbb{R} \to \mathbb{R}$

$$x \mapsto \begin{cases} 0 \text{ if } x \leq 0 \\ x^2 \text{ otherwise} \end{cases}$$



Registration of a trimmed source (blue) onto a richer target (red)

Partial Matching



*Weighted-Localized version : **definition***

Partial matching dissimilarity function (localized, normalized):

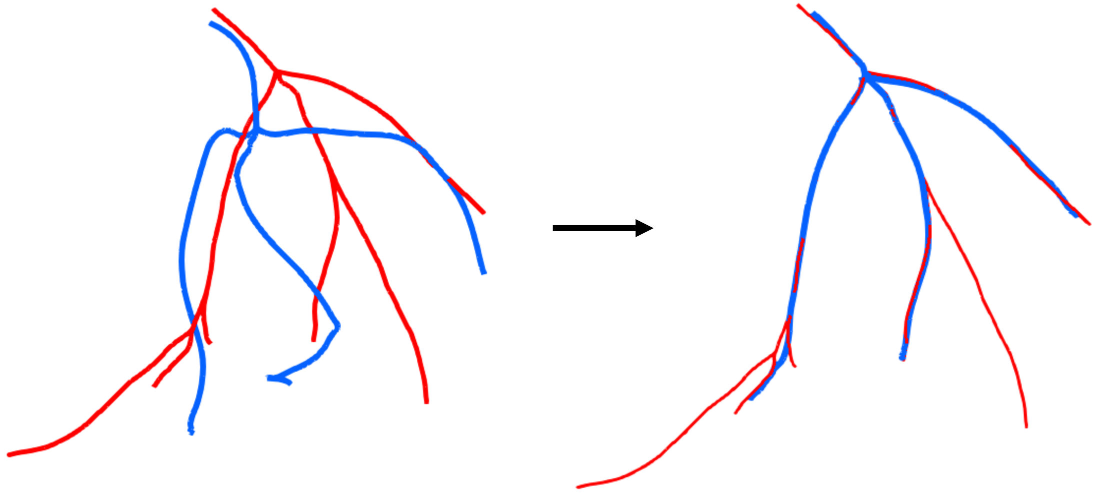
$$\boldsymbol{A}_2(S,T) = \int_S g\big(\omega_S(\boldsymbol{x}) - \widetilde{\omega}_T(\boldsymbol{x})\big)dx$$

With :

$$\widetilde{\omega}_T(\boldsymbol{x}) = \int_T \min\left(1, \frac{\omega_S(\boldsymbol{x})}{\omega_T(\boldsymbol{y})}\right) k(\boldsymbol{x},\boldsymbol{y})dy\,,$$

$$g : \mathbb{R} \to \mathbb{R}$$

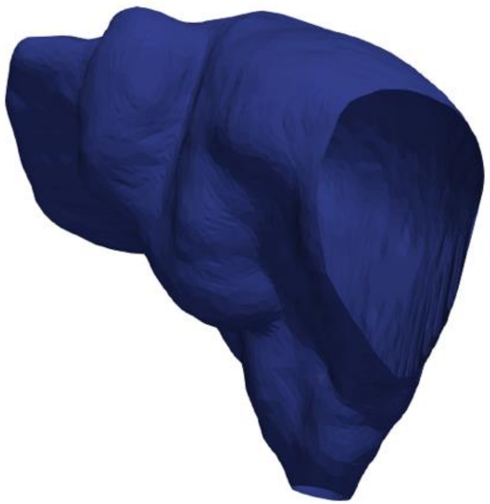
$$\mathrm{x} \mapsto \begin{cases} 0 \; if \; x \leq 0 \\ x^2 \; otherwise \end{cases}$$



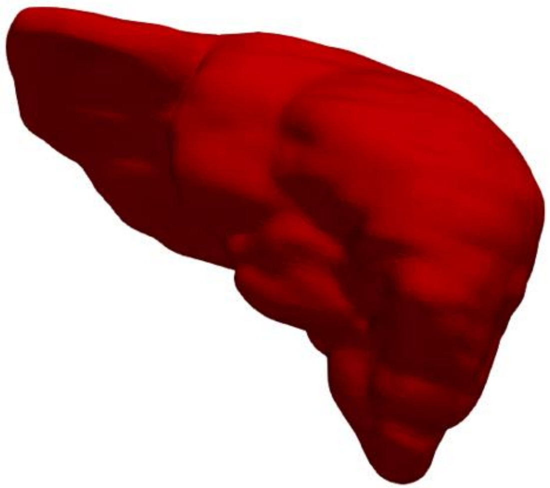
Registration of a trimmed source (blue) onto a richer target (red), **Data attachment term A_2**

Partial Matching

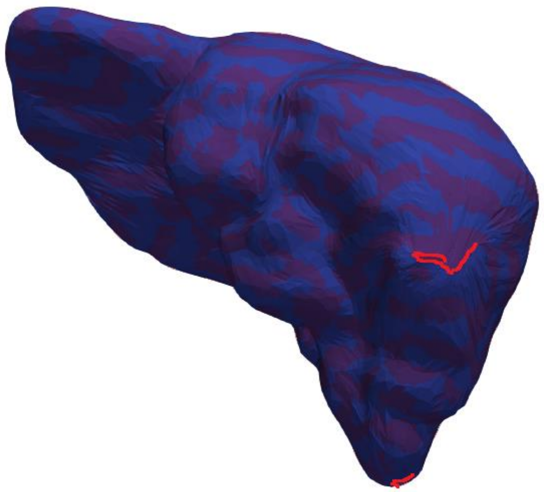
Application to Livers Surfaces



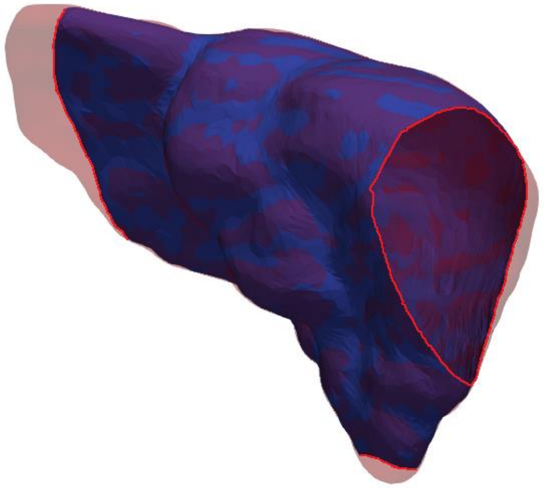
Source (CBCT)



Target (CT)



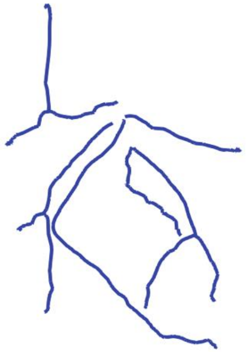
Registration with classic varifolds data attachment



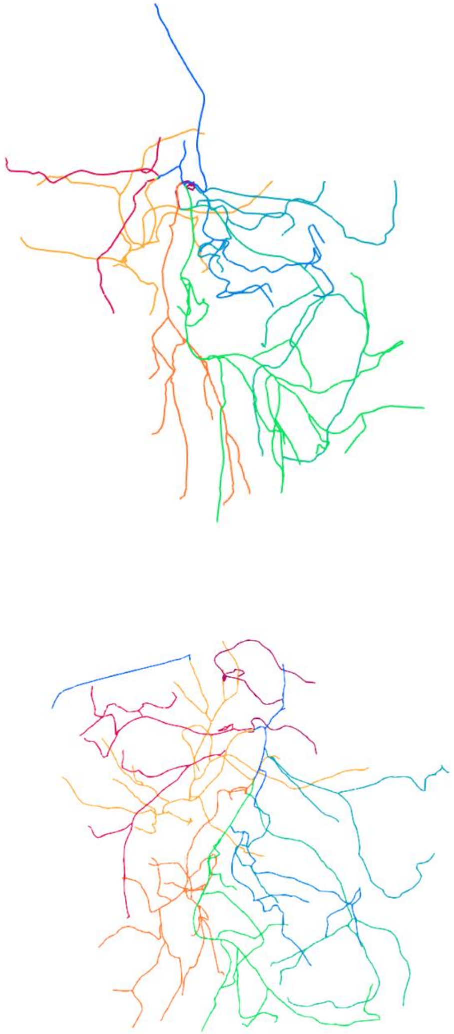
Registration with local, normalized, partial matching

Applications

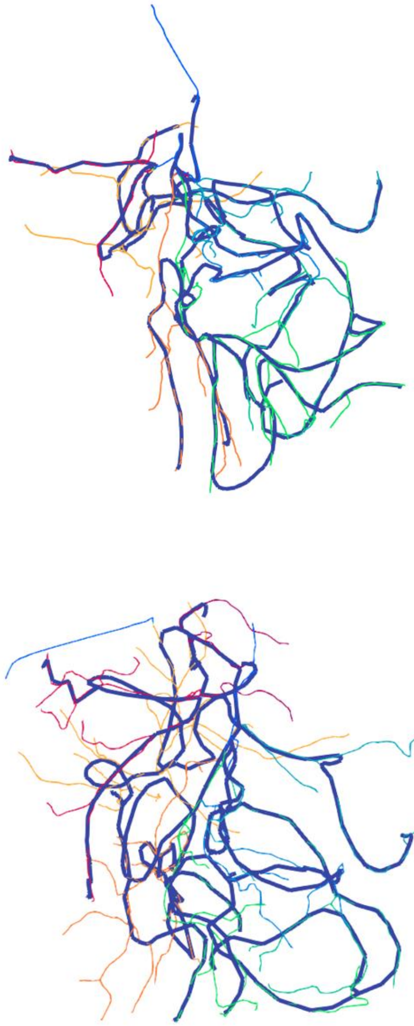
Application to Vascular Trees



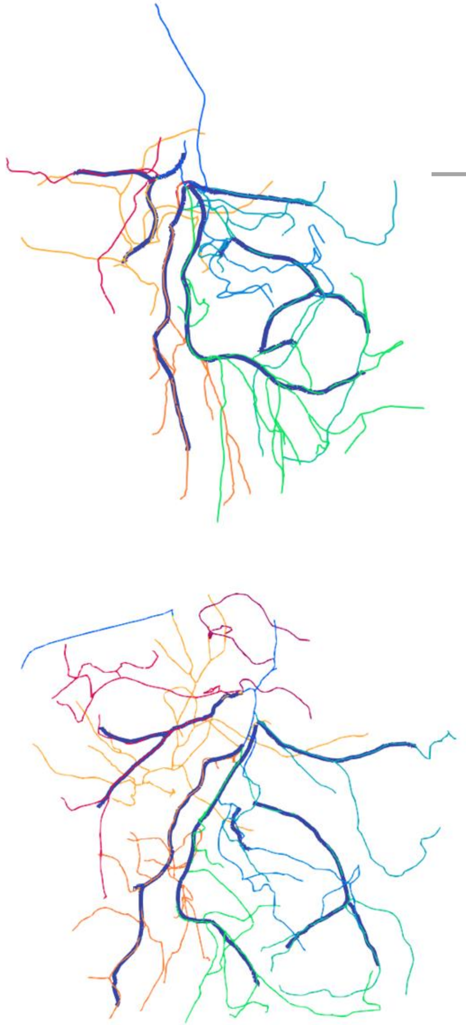
Source : set of arteries manually selected that could be matched to target.



Targets (new subjects)



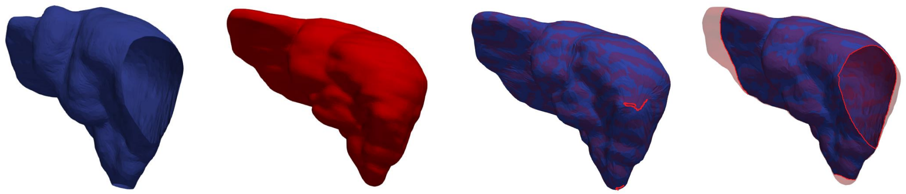
Registration with classic varifolds data attachment



Registrations with local, normalized, partial matching

Partial Matching in the Space of Varifolds

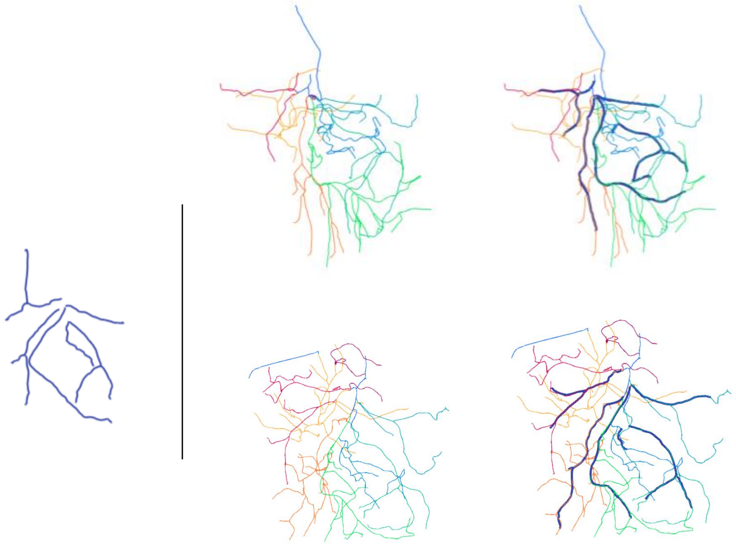
Antonsanti, Glaunès, Benseghir, Jugnon, Kaltenmark



Take-home message

LDDMM and Partial Matching ? Possible !

Adapted to different shapes just like varifolds and cie.



To go further …

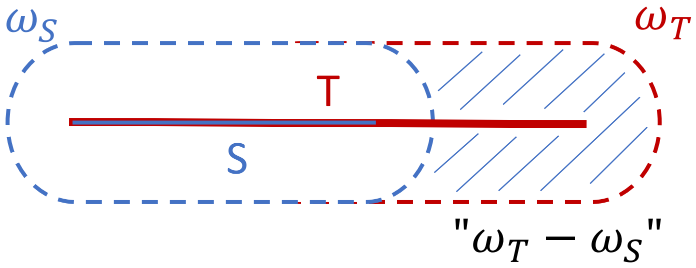
We can also include the target into the source…

More generic framework ? [Bronstein 2009]

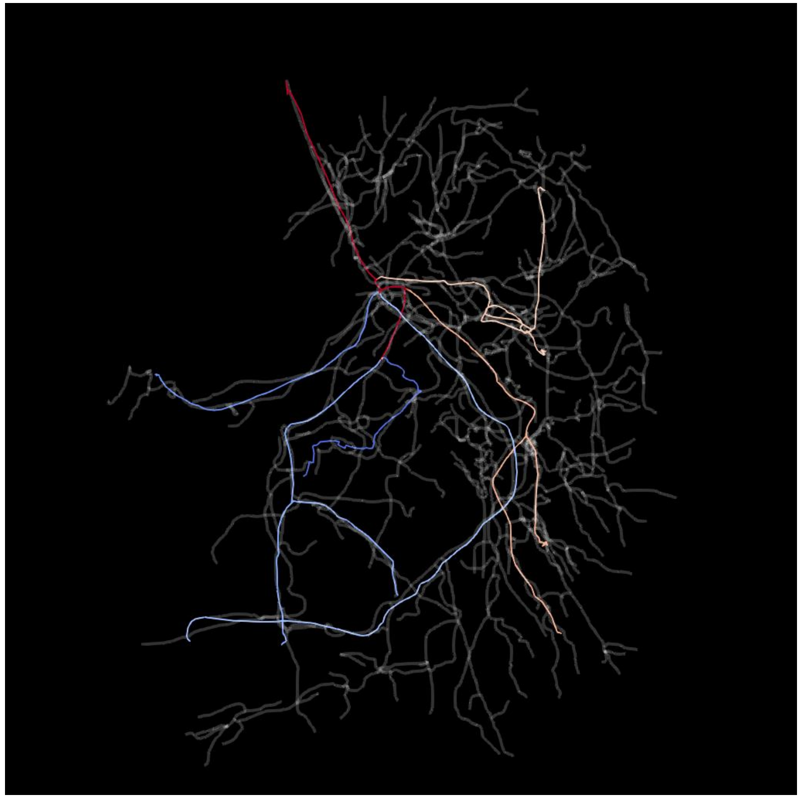
Improve the normalization.

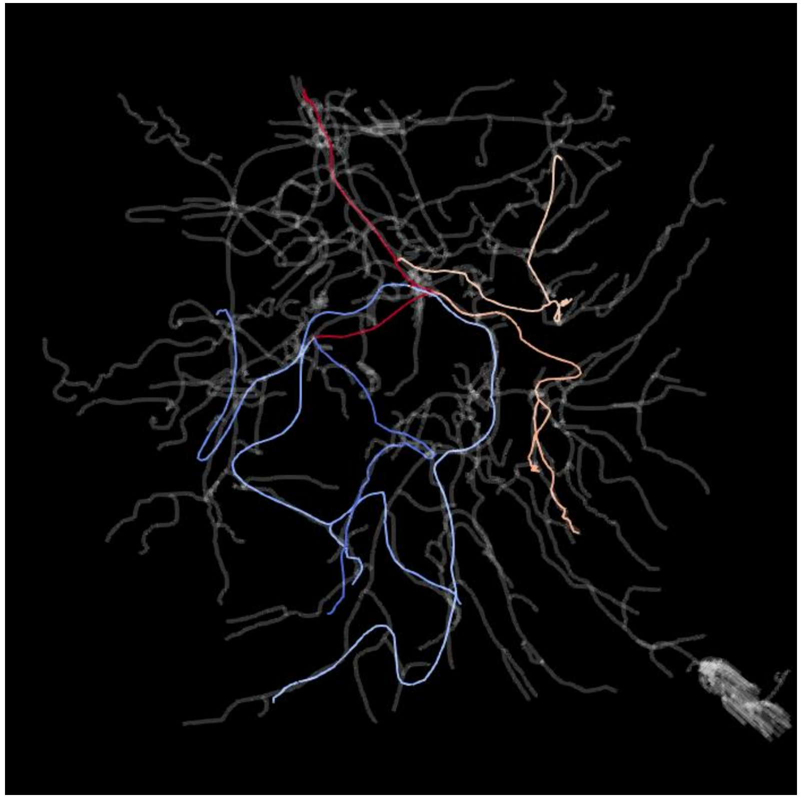
Contact : pierrelouis.antonsanti@ge.com

Partial Matching



Toward real world applications





Registration with classic varifolds data attachment, with the best regularization setting observed.