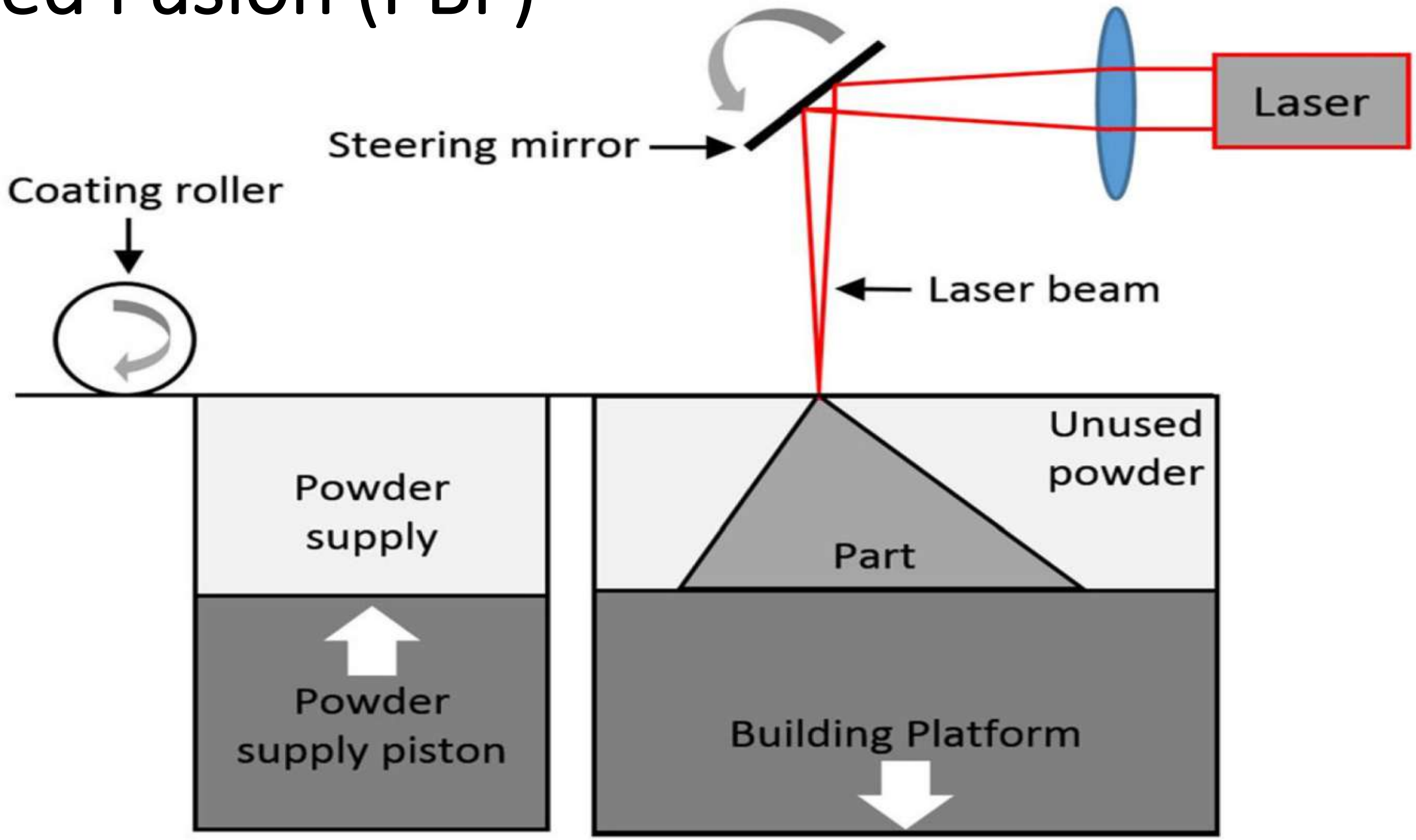
Trajectory optimization for powder bed fusion additive manufacturing

Mathilde Boissier,

PhD advisors: Grégoire Allaire, Christophe Tournier

In collaboration with Tonia Maria ALAM

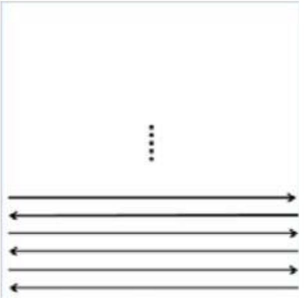
Powder Bed Fusion (PBF)



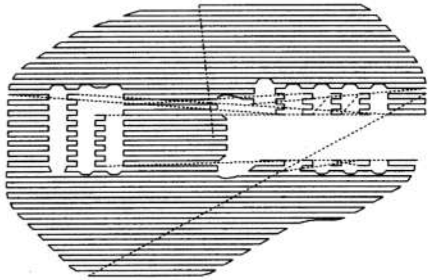
Bikas, H., P. Stavropoulos, and G. Chryssolouris, *Additive Manufacturing Methods and Modelling Approaches: A Critical Review* In: The International Journal of Advanced Manufacturing Technology 83.1-4 (2016), pp. 389–405.

https://www.youtube.com/watch?v=yiUUZxp7bLQ

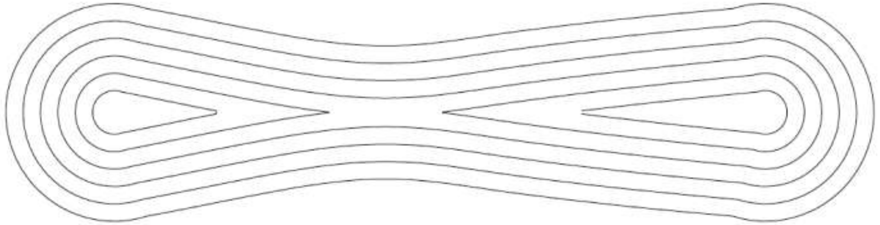
Notion of “good path”[a]



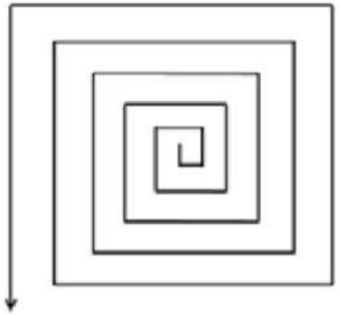
Parallel



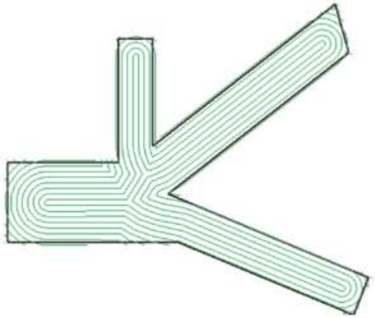
Zigzag



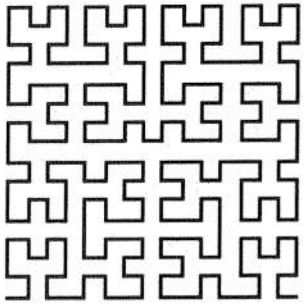
Contour



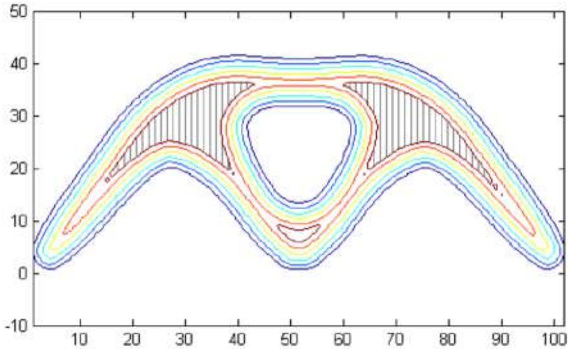
Spiral



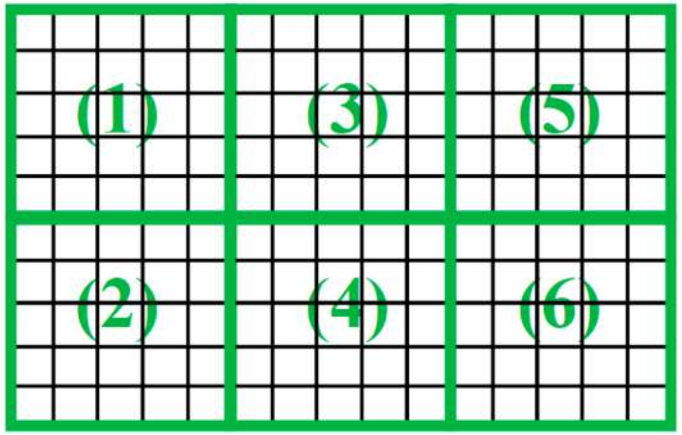
Medial Axis Transform



Fractal



Hybrid



(a) D. Ding, Z. Pan, D. Cuiuri, H. Li, and S. van Duin, *Advanced design for additive manufacturing: 3d slicing and 2d path planning*, New trends in 3d printing, (2016), pp. 1–23.

Objectives of this work

How to use shape optimization to facilitate the generation of “good” scanning path?

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- Optimization of the path “from scratch”
- (Concurrent optimization between part shape and scanning path)

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Bibliography:

- T.M. ALAM, Some optimal control problem of partial differential equations and applications to the selective laser melting process (SLM), PhD thesis, Université Polytechnique Hauts-de-France, 2020

- Q. Chen, J. Liu, X. Liang, and A. C. To, A level-set based continuous scanning path optimization method for reducing residual stress and deformation in metal additive manufacturing, Computer Methods in Applied Mechanics and Engineering, 360 (2020), p. 112719.

Overview

- Modelling assumptions
- Scanning path optimization
- Modifying the path topology
  - Physical approach: coupling scanning path and power optimization
  - Topology approach: topological optimization of the scanning path

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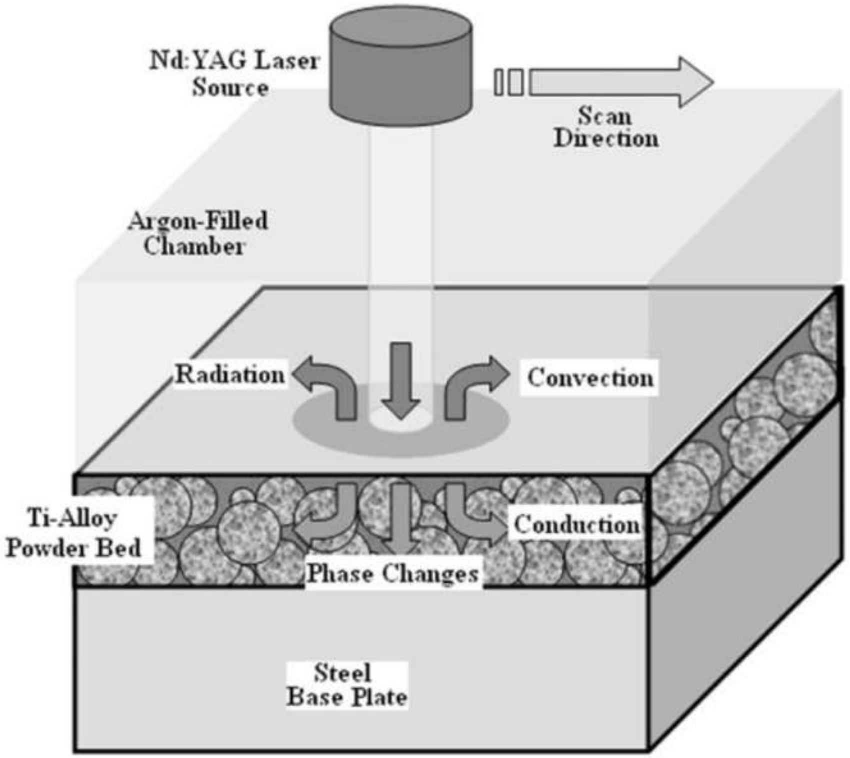
Modelling the scanning process [(a)]



Microscale modelling:

- accurate model for the change of state and melting pool
- 4 states considered: powder, solid, liquid and gaseous

Macroscale modelling:

- simplified model without accurate computation of the change of state and melting pool
- 2 states considered: powder, solid

(a) T. DebRoy, H. Wei, J. Zuback, T. Mukherjee, J. Elmer, J. Milewski, A. M. Beese, A. Wilson-Heid, A. De, and W. Zhang, *Additive manufacturing of metallic components–process, structure and properties*, Progress in Materials Science, 92 (2018), pp. 112–224.

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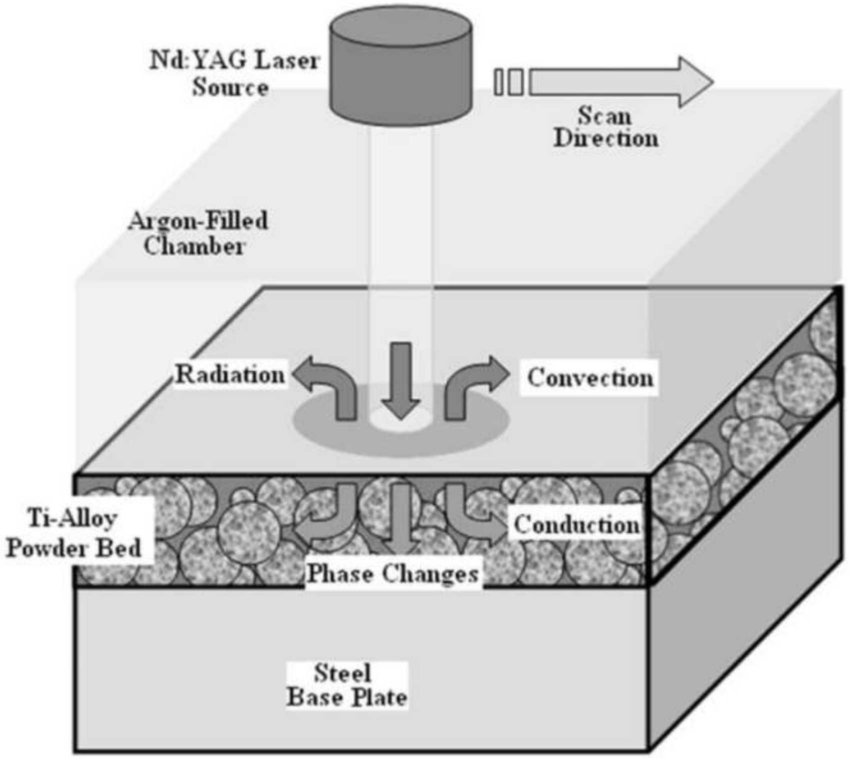

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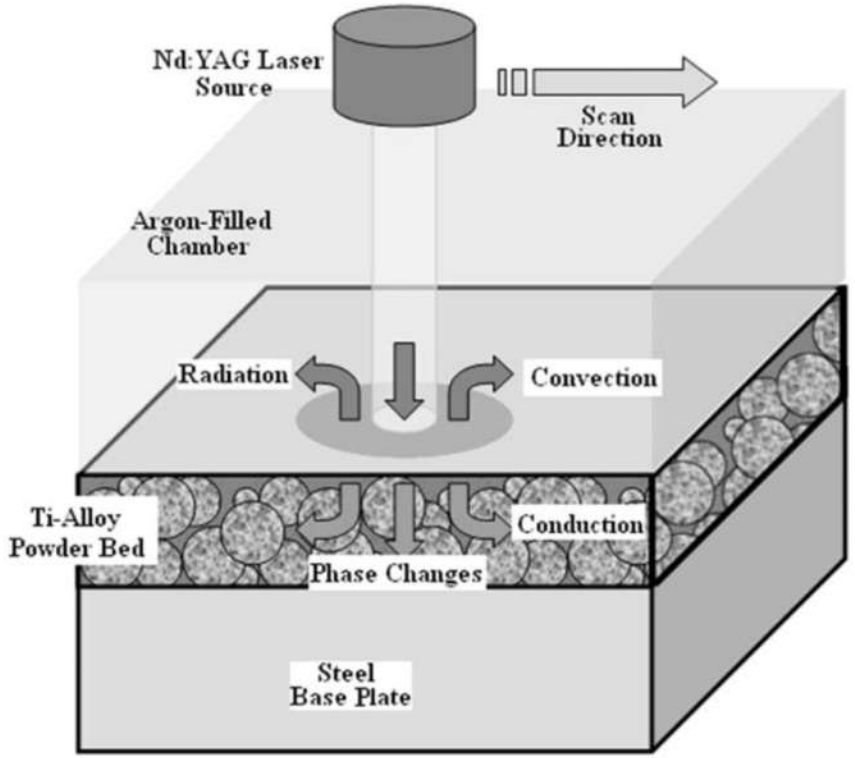

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STAKES AT A MACROSCOPIC SCALE

- **thermo-mechanics:** thermal expansion, residual stresses, solidification of a layer
- **kinematics:** minimal execution time

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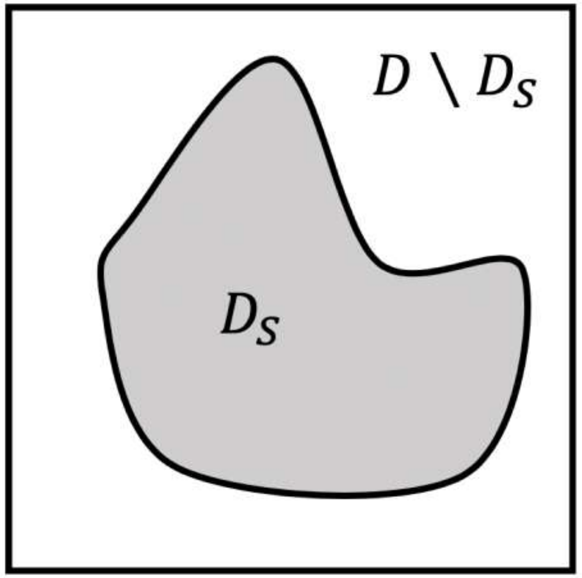
Steady model

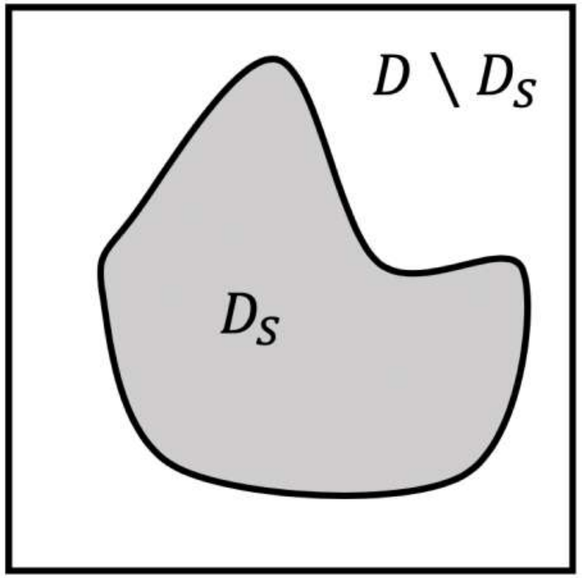
The whole source is switched on at once



Steady model

The whole source is switched on at once

Dirac function of the path $\boldsymbol{\Gamma}$: $q = P\chi_{\Gamma}$



Steady model

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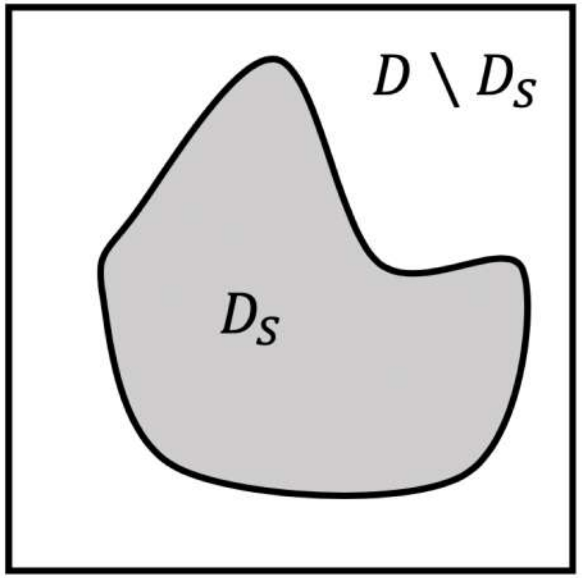
Dirac function of the path $\mathbf{\Gamma}$: $q = P\chi_\Gamma$

Temperature equation:
$$\begin{cases} -\nabla(\lambda\nabla y) + \beta(y - y_{ini}) = P\chi_\Gamma, & x \in D \\ \lambda\partial_n y = 0, & x \in \partial D \end{cases}$$



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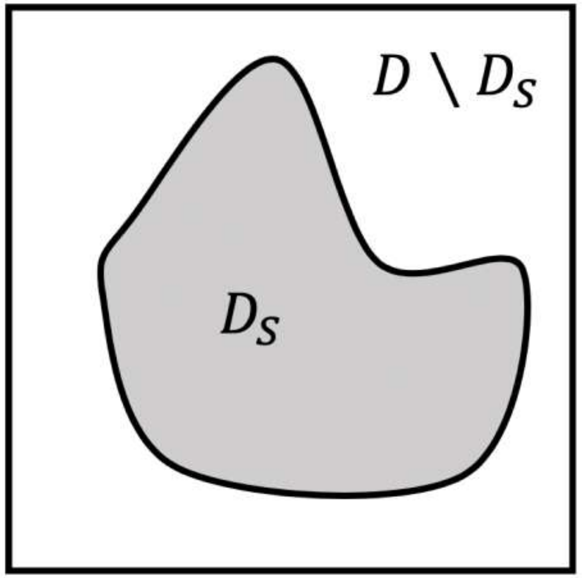
Optimization problem:

$$\min L_F = |\Gamma| \quad s.t. \begin{cases} \forall x \in D_S, & y(x) \geq y_\phi \\ \forall x \in D \setminus D_S, & y(x) \leq y_{M,out} \\ \forall x \in D_S, & y(x) \leq y_{M,in} \end{cases}$$



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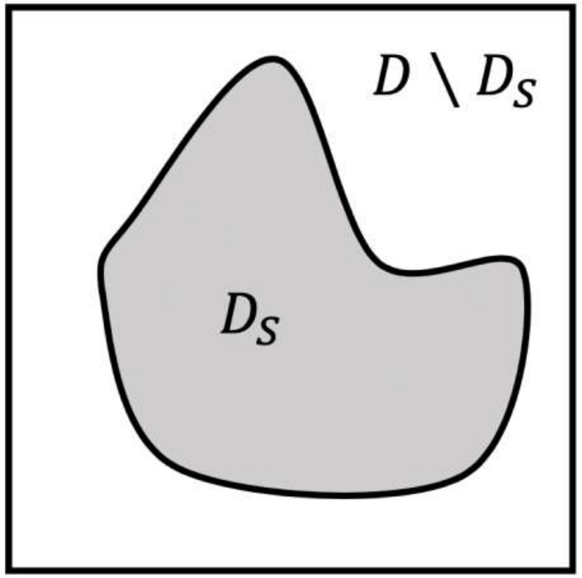

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Dirac function of the path **Γ**: $q = P\chi_\Gamma$

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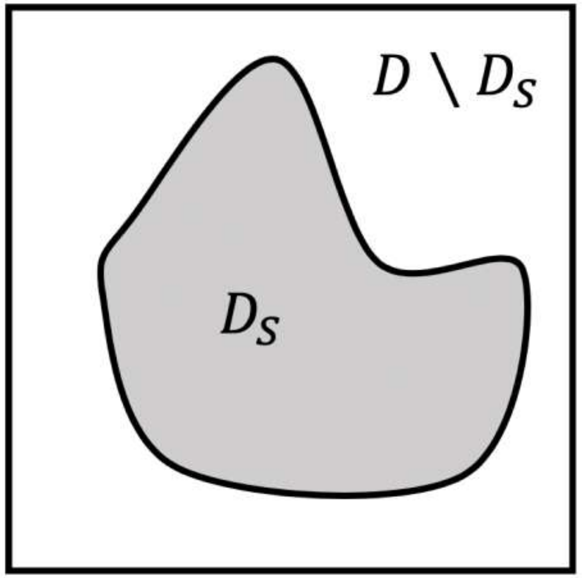


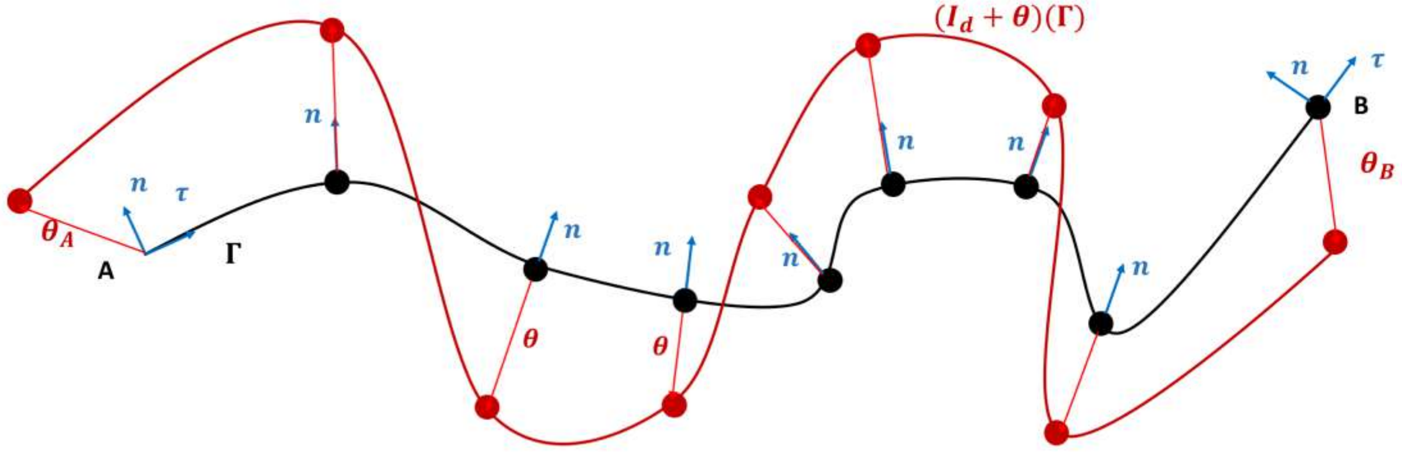
Augmented Lagrangian method : $\mathcal{L}_{ALM}(\Gamma, l_C; \mu_C) = |\Gamma| + l_C C + \frac{\mu_C}{2} C^2 + \chi_{\Gamma\subset D}, \quad C = C_\Phi + C_{M,out} + C_{M,in}$

Overview

- Modelling assumptions
- Scanning path optimization
- Modifying the path topology
    - Physical approach: coupling scanning path and power optimization
    - Topology approach: topological optimization of the scanning path

Optimization algorithm

Gradient computation: shape differentiation theory (Differentiate and then discretize)



Γ regular curve with chosen orientation, tangent τ, normal n, curvature κ and endpoints A and B.

A. Henrot and M. Pierre, *Shape variation and optimization*, EMS Tracts in Mathematics, European Mathematical Society (EMS), Zürich, 2018.

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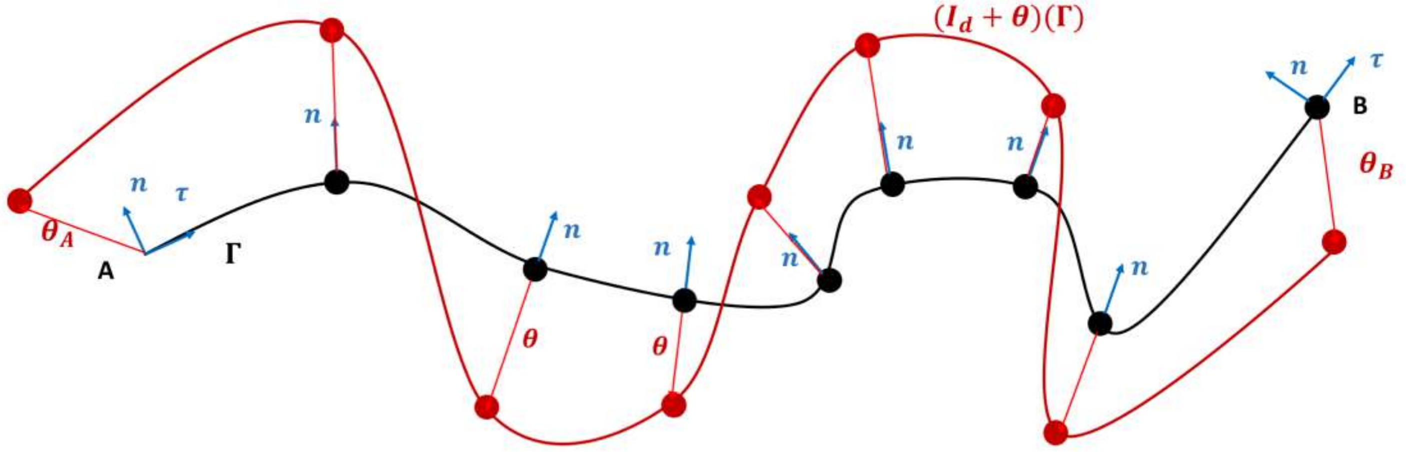

Γ regular curve with chosen orientation, tangent τ, normal n, curvature κ and endpoints A and B.

Shape derivative of $J(\Gamma) = \int_\Gamma f(s)ds$: $\quad DJ(\Gamma)(\theta) = \int_\Gamma (\partial_n f + \kappa f)\theta \cdot n ds + f(B)\theta(B)\cdot\tau(B) - f(A)\theta(A)\cdot(A)$

Gradient descent:

$$J(\Gamma^{n+1}) = J(\Gamma^n) + DJ(\Gamma^n)(\theta) + o(\theta)$$

θ chosen such that $J(\Gamma^{n+1}) \leq J(\Gamma^n)$

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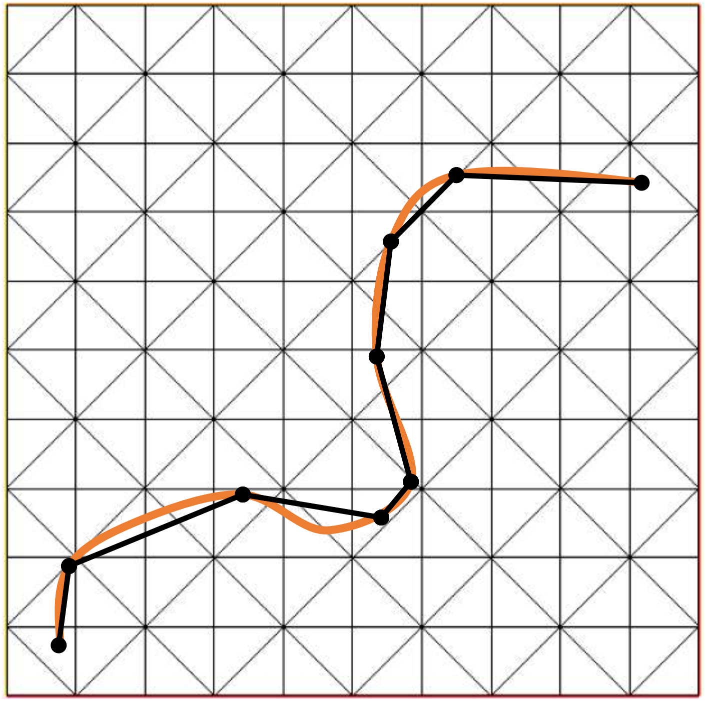
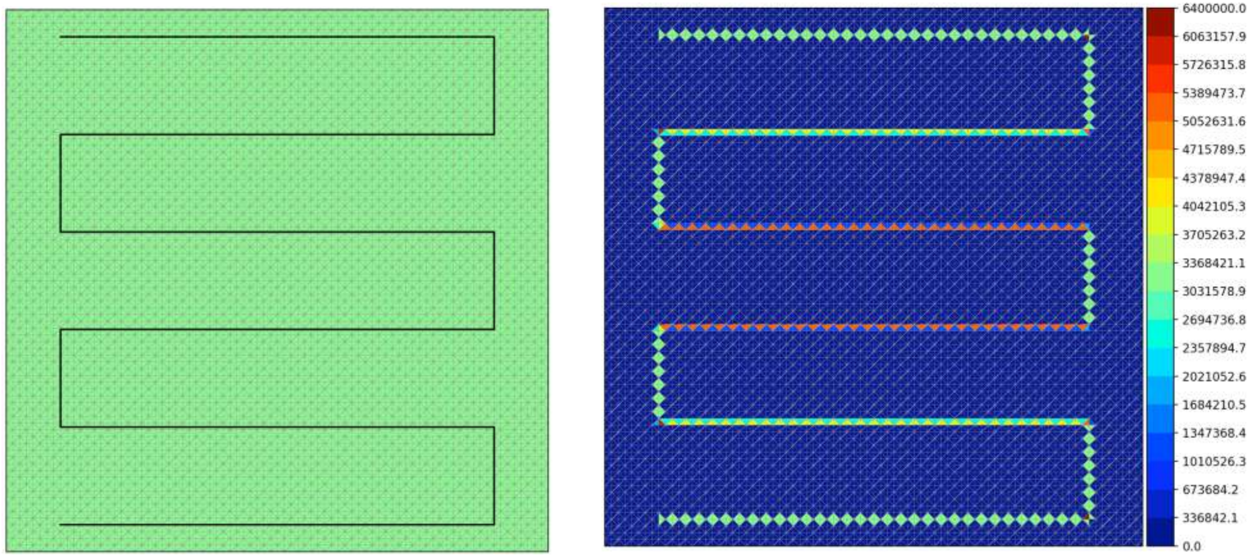
Numerical representation: front-tracking

Fixed physical mesh – Path described by a **broken line** **defined by nodal points**



Crucial points:

- Control the broken line definition
- Information mapping from the broken line to the physical mesh and vice versa



Optimization algorithm

1. Initialization of the path Γ^0

2. Heat equation resolution and computation of the objective function, constraints

3. Computation of the gradient

4. For $k \leq N_{MAX}$:

   a) Update the path Γ^k to a new path Γ^{k+1} using the gradients with a descent step

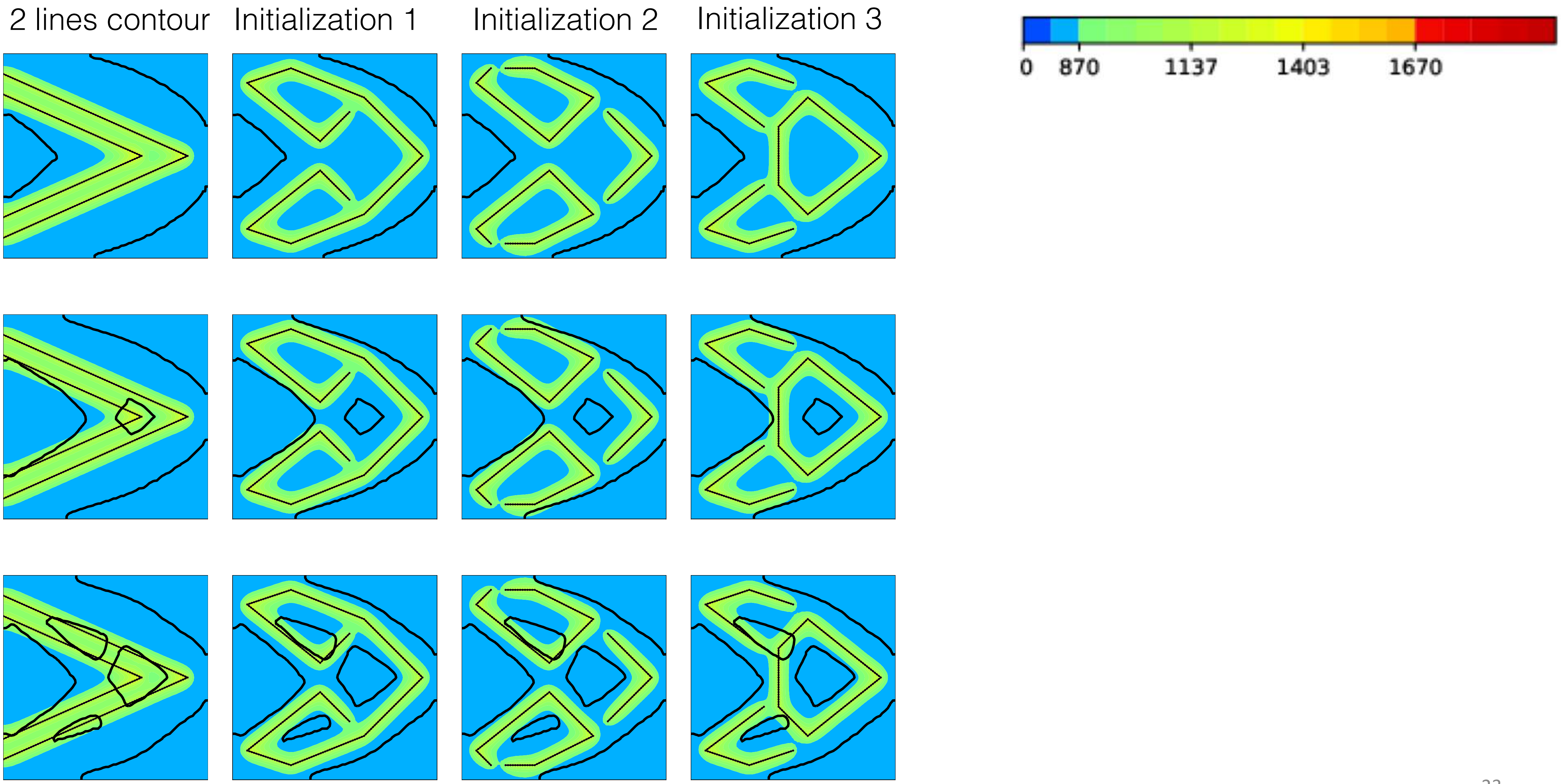
   b) Re-discretize the path Γ^{k+1} to maintain its coherence

   c) Heat equation resolution and computation of the objective function, constraints

   d) If the merit function is improved:
      Iteration accepted

   e) Else:
      Iteration rejected: descent step decreased

Different initializations – Aluminum ($\lambda = 130 W m^{-1} K^{-1}$)



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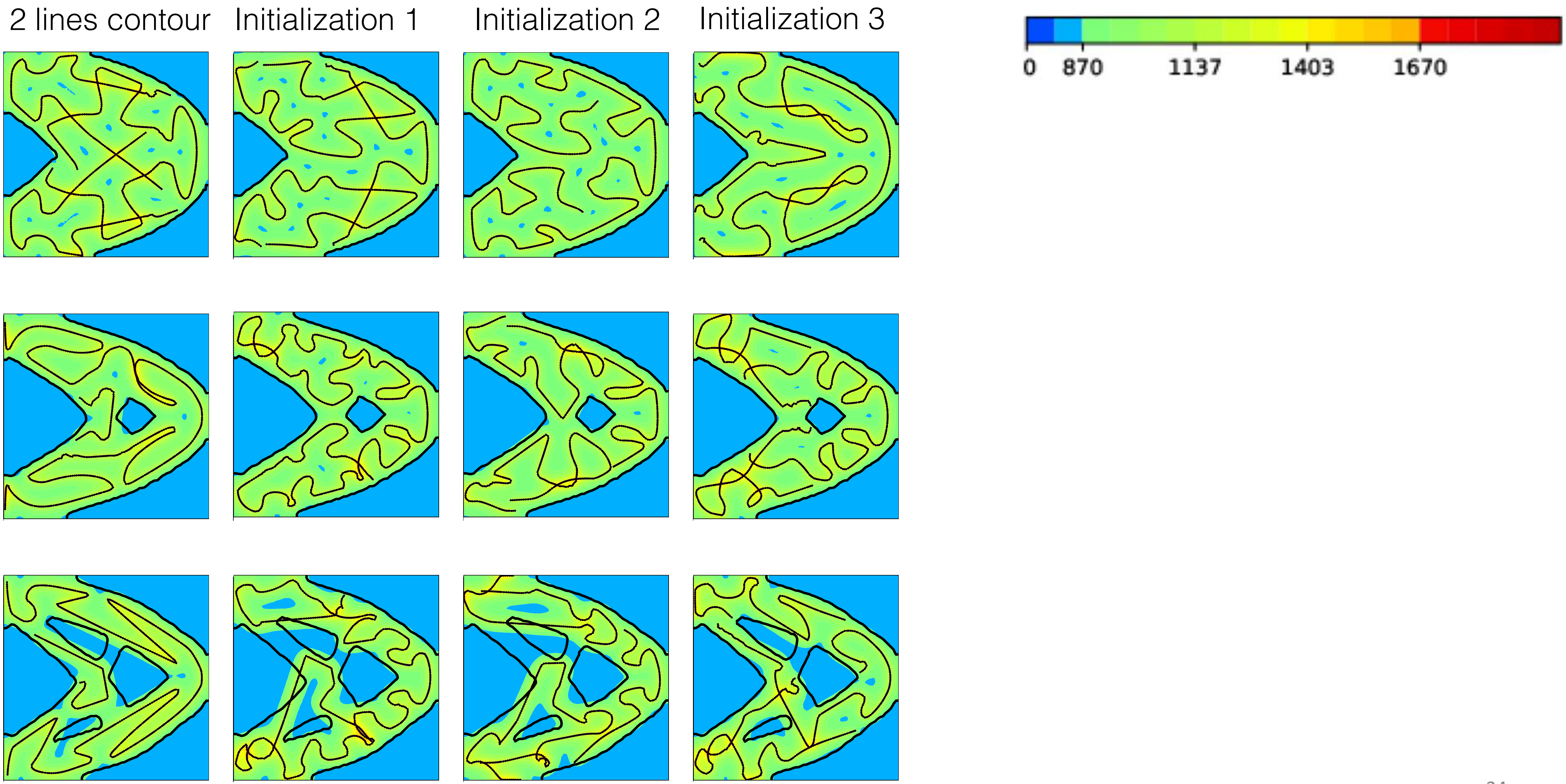

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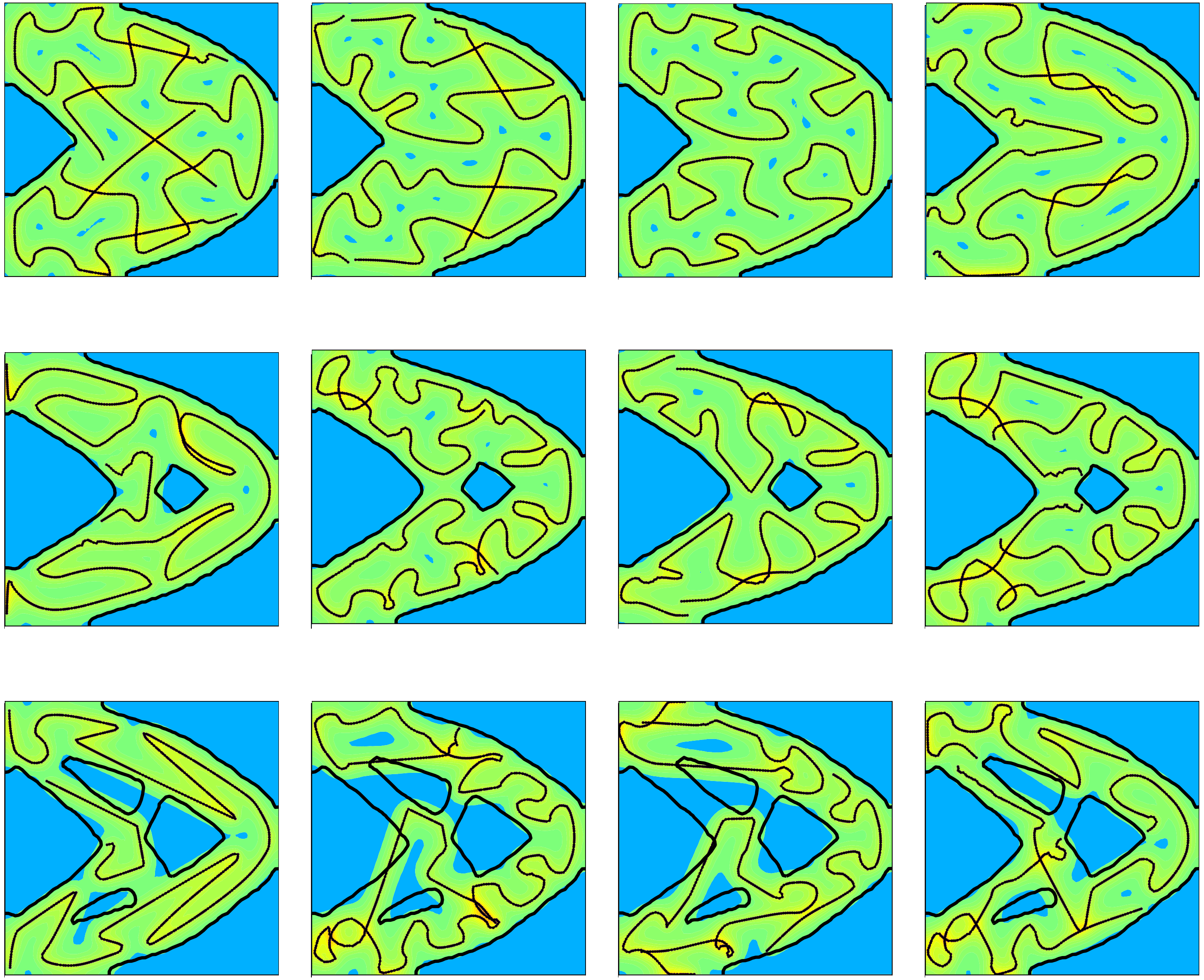
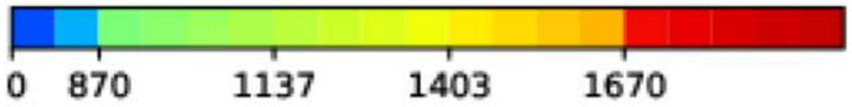
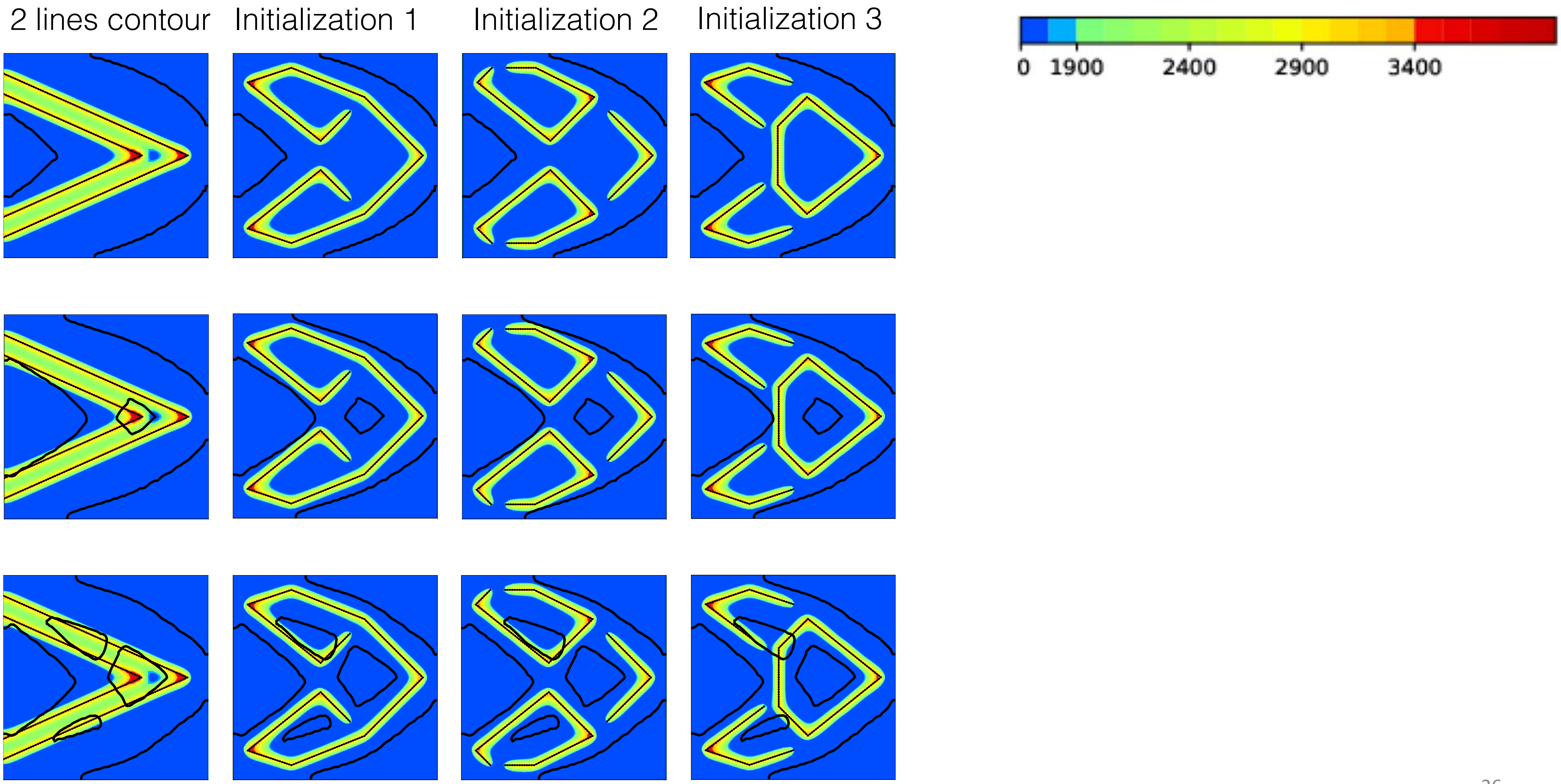




- Results really dependent on the initialization
- Correct adaptation to the shape if allowed by the conductivity => **shape thickness**

Different initializations – Titanium ($\lambda = 15 W m^{-1} K^{-1}$)



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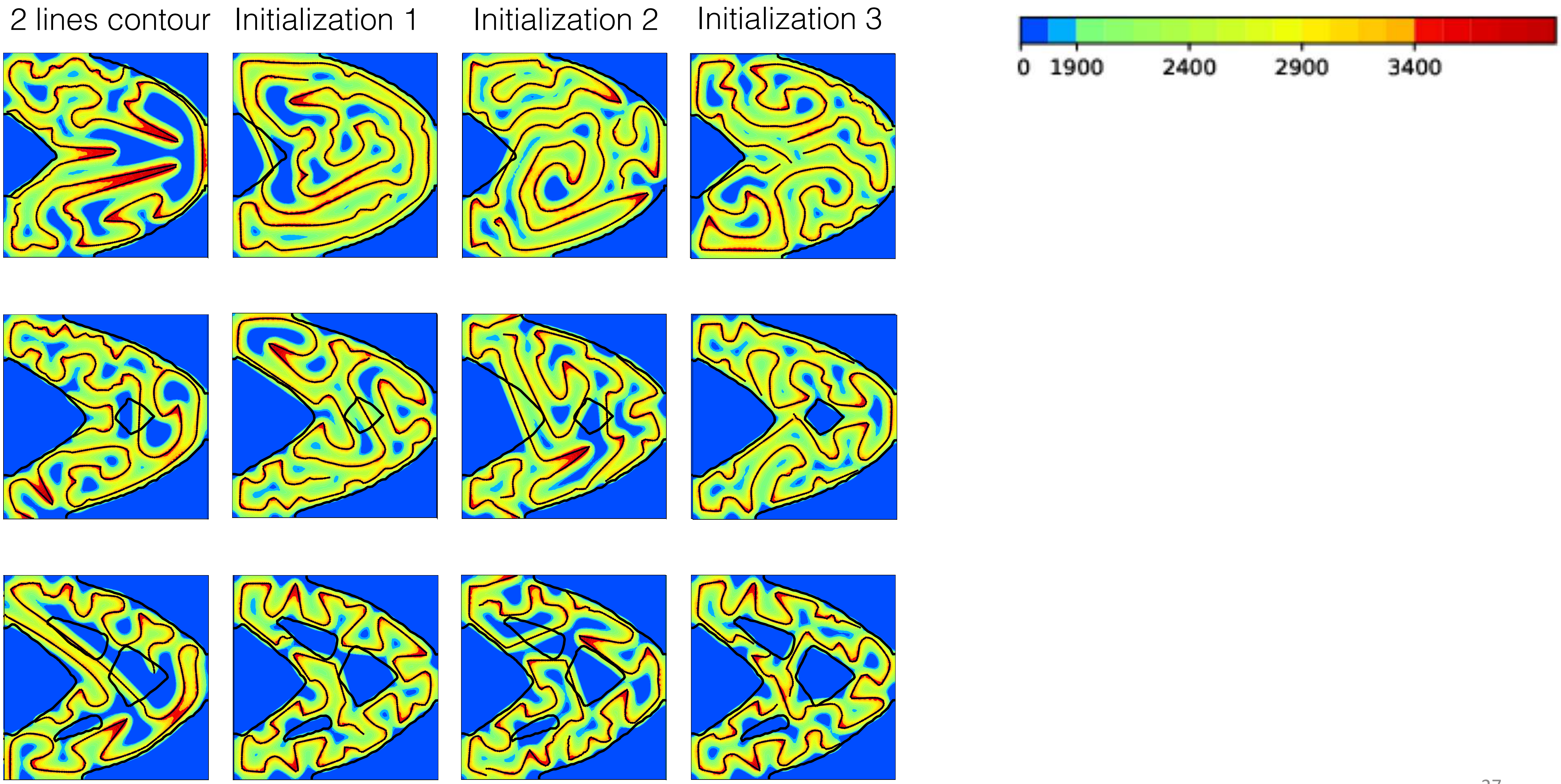

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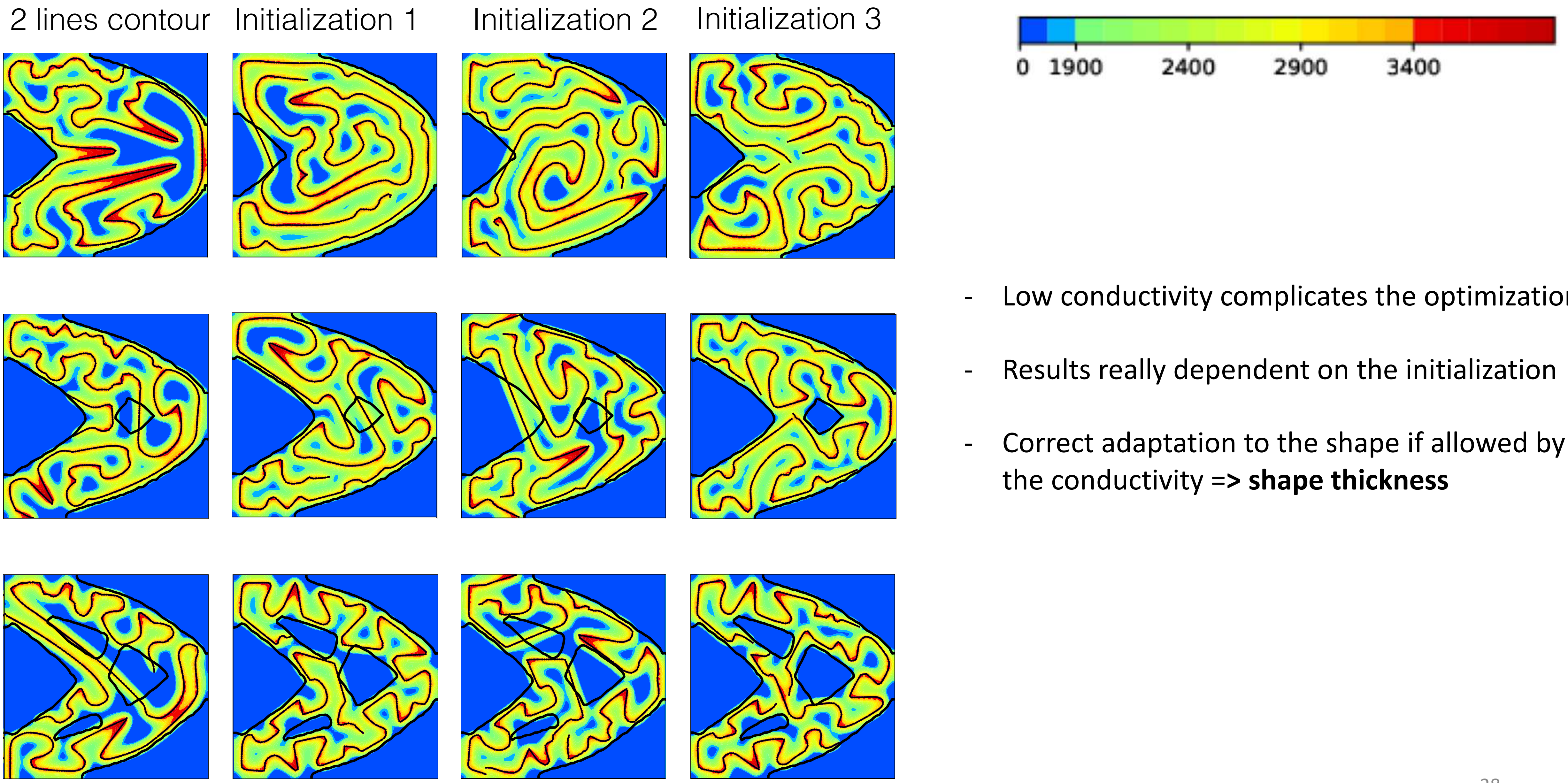

- Low conductivity complicates the optimization
- Results really dependent on the initialization
- Correct adaptation to the shape if allowed by the conductivity => **shape thickness**

Overview

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Steady model – including power

Dirac function of the path $\mathbf{\Gamma}$: $q = P\zeta\chi_{\Gamma}$, $\zeta: s \in [0, L] \mapsto \zeta(s) \in \{0,1\}$, $\zeta \in L^2([0, L], \{0,1\})$

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Optimization problem:

$$\min_{\Gamma,\zeta} L_F = |\Gamma| = \int_\Gamma ds \quad s.t. \begin{cases} C_\phi(\Gamma) = \int_{D_S} \left[\left(y_\phi - y\right)^+\right]^2 dx = 0 \\ C_{M,out}(\Gamma) = \int_{D \setminus D_S} \left[\left(y - y_{M,out}\right)^+\right]^2 dx = 0 \\ C_{M,in}(\Gamma) = \int_{D_S} \left[\left(y - y_{M,in}\right)^+\right]^2 dx = 0 \end{cases}$$

Augmented Lagrangian method :

$$\mathcal{L}_{ALM}(\Gamma, \zeta, l_C; \mu_C) = |\Gamma| + l_C C + \frac{\mu_C}{2} C^2 + \chi_{\zeta\in\{0,1\}} + \chi_{\Gamma\subset D}, \ C = C_\Phi + C_{M,out} + C_{M,in}$$

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Steady model – relaxation-penalization method

Relaxation :

$$\zeta : s \in [0, L] \mapsto \zeta(s) \in [0,1], \qquad \zeta \in L^2([0, L], [0,1])$$

Steady model – relaxation-penalization method

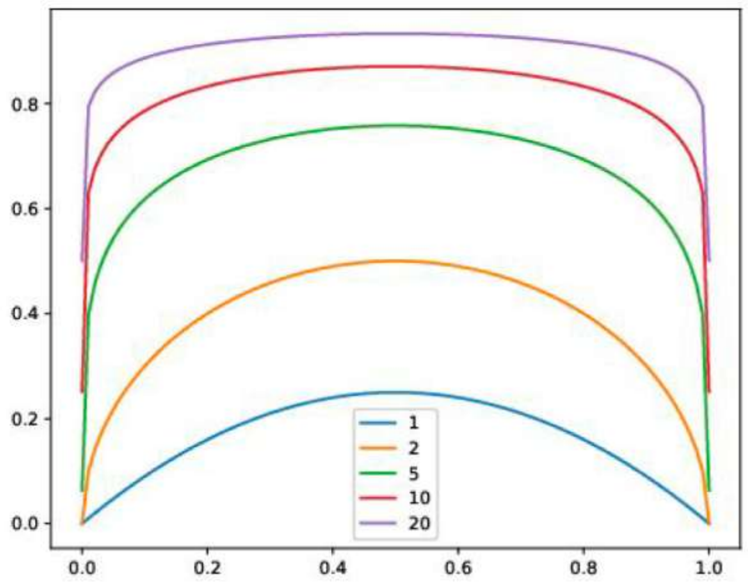
Relaxation :

$$\zeta : s \in [0, L] \mapsto \zeta(s) \in [0,1], \qquad \zeta \in L^2([0, L], [0,1])$$

Penalization :

$$\mathcal{P} = \frac{1}{|L|} \int_\Gamma f_{pen}\big(\zeta(s)\big) ds$$

$$f_{pen} : \xi \in [0,1] \mapsto \xi(1-\xi)^{q_{pen}}, \qquad q_{pen} = 1$$



Steady model – relaxation-penalization method

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Temperature equation:

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$$\min_{\Gamma,\zeta} L_F = |\Gamma| = \int_\Gamma ds \quad s.t. \quad \begin{cases} C_\phi(\Gamma,\zeta) = \int_{D_S} \left[\left(y_\phi - y\right)^+\right]^2 dx = 0 \\ C_{M,out}(\Gamma,\zeta) = \int_{D\setminus D_S} \left[\left(y - y_{M,out}\right)^+\right]^2 dx = 0 \\ C_{M,in}(\Gamma,\zeta) = \int_{D_S} \left[\left(y - y_{M,in}\right)^+\right]^2 dx = 0 \end{cases}$$

And

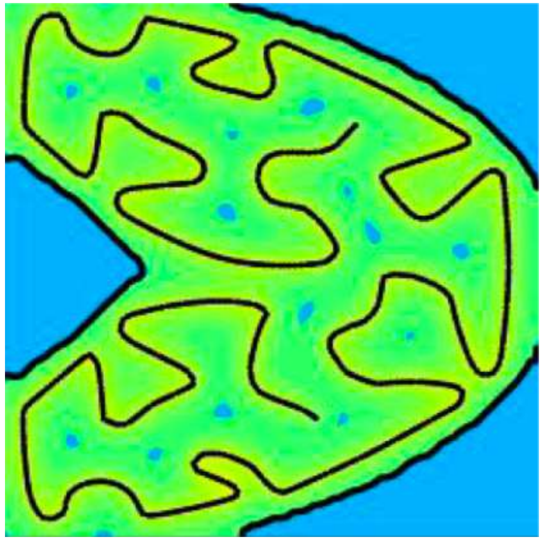
$$\mathcal{P}(\Gamma,\zeta) = \frac{1}{|L|}\int_\Gamma f_{pen}\big(\zeta(s)\big)ds = 0$$

Augmented Lagrangian method :

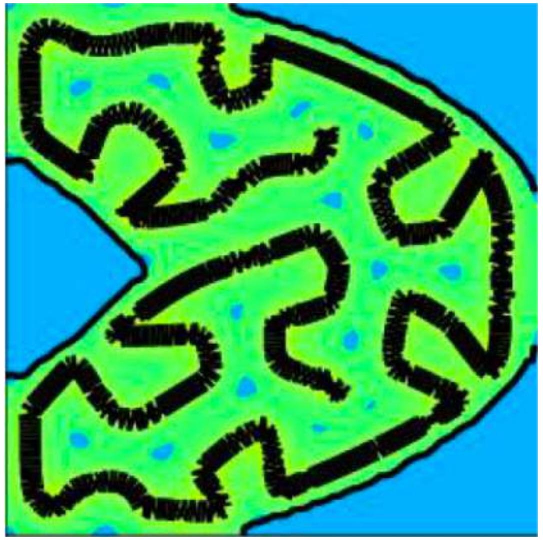
$$\mathcal{L}_{ALM}(\Gamma,\zeta,l_C,l_P;\mu_C,\mu_P) = |\Gamma| + l_C C + \frac{\mu_C}{2}C^2 + l_P\mathcal{P} + \frac{\mu_P}{2}\mathcal{P}^2 + \chi_{\zeta\in[0,1]} + \chi_{\Gamma\subset D}$$

Steady model – relaxation-penalization method

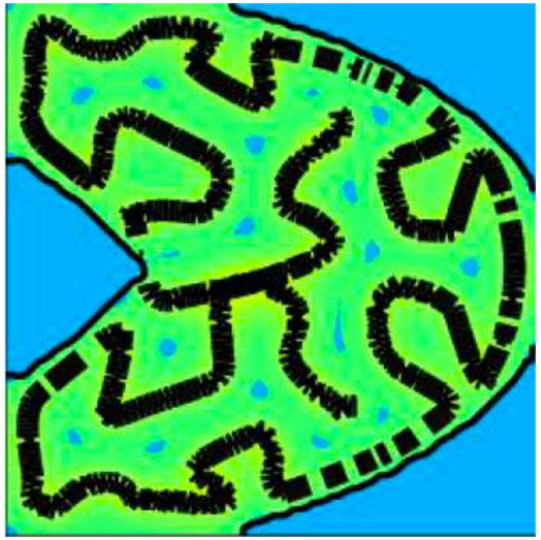
$\zeta_{ini} = 0.5$



Path only optimization



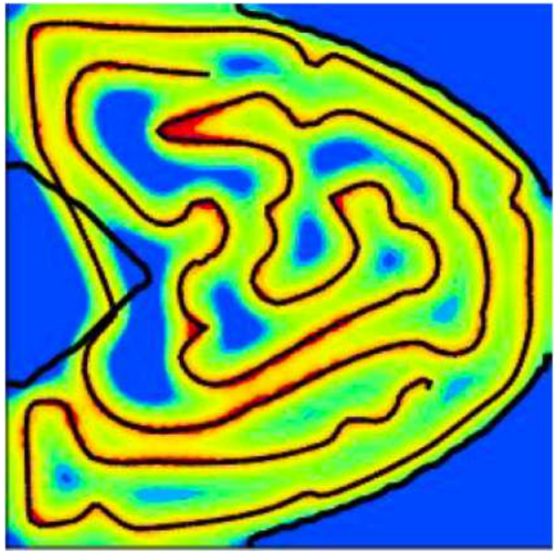
Path and power optimization
Relaxed problem



Path and power optimization
Relaxed-penalized problem

Steady model – relaxation-penalization method

$\zeta_{ini} = 0.5$



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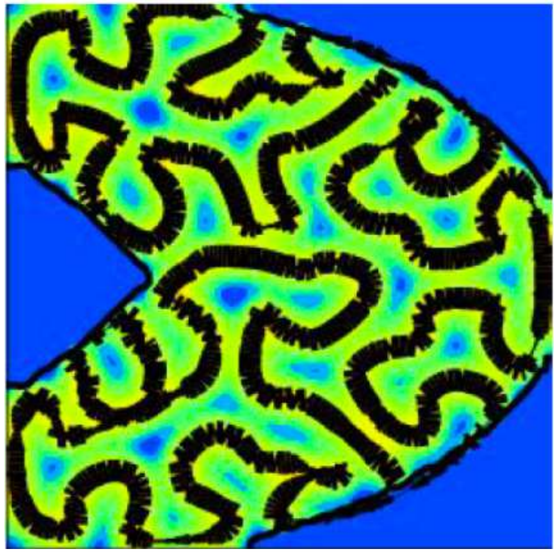
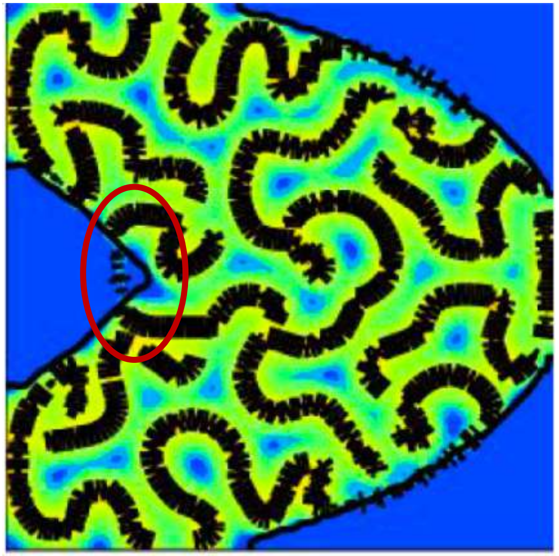


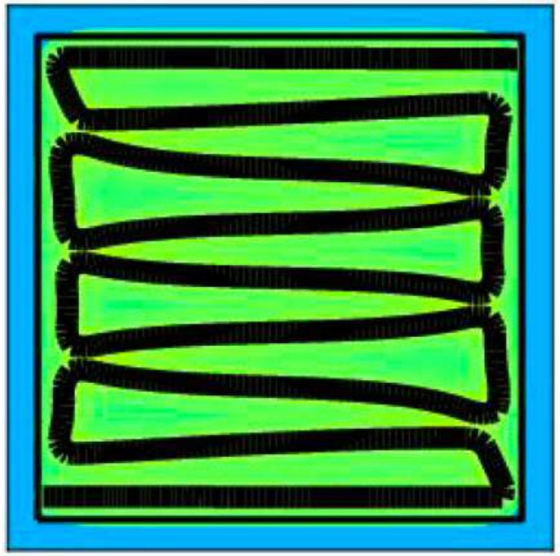
Path and power optimization
Relaxed problem



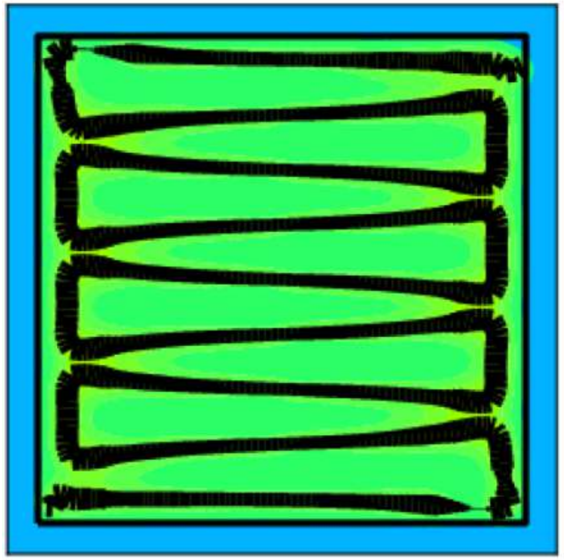
Path and power optimization
Relaxed-penalized problem

Steady model – relaxation-penalization method

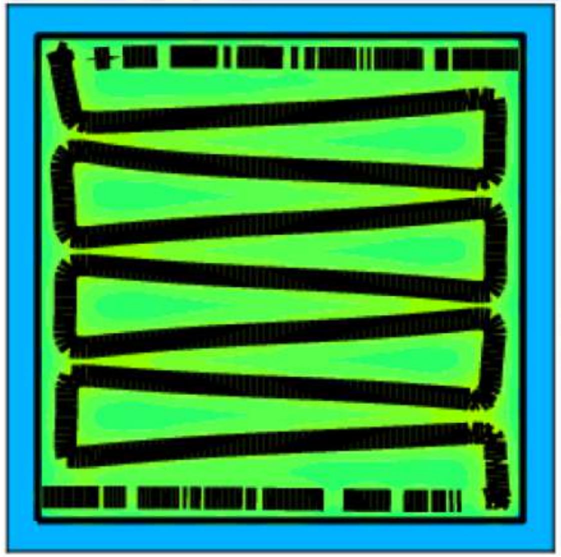
$\zeta_{ini} = 0.5$



Path only optimization



Path and power optimization
Relaxed problem



Path and power optimization
Relaxed-penalized problem

Steady model – total variation penalization

Controlling the power jumps ➡ Denoising problem $^{(\boldsymbol{a})}$

(a) A. Chambolle, V. Caselles, D. Cremers, M. Novaga, and T. Pock, An introduction to total variation for image analysis, Theoretical foundations and numerical methods for sparse recovery, 9 (2010), p. 227.

Steady model – total variation penalization

Controlling the power jumps ➡ Denoising problem [a]

Augmented Lagrangian method :

$$\mathcal{L}_{ALM}(\Gamma, \zeta, l_C, l_P; \mu_C, \mu_P) = |\Gamma| + l_C C + \frac{\mu_C}{2} C^2 + l_P \mathcal{P} + \frac{\mu_P}{2} \mathcal{P}^2 + \chi_{\zeta \in [0,1]} + \chi_{\Gamma \subset D} + l_{TV} \frac{1}{|\Gamma|} \int_{\Gamma} |d_s \zeta| ds$$

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Steady model – total variation penalization

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Proximal gradient algorithm

$$\mathcal{L}_{ALM}(\Gamma, \zeta, l_C, l_P; \mu_C, \mu_P) = |\boldsymbol{\Gamma}| + \boldsymbol{l_C C} + \frac{\boldsymbol{\mu_C}}{\mathbf{2}} \boldsymbol{C^2} + \boldsymbol{l_P \mathcal{P}} + \frac{\boldsymbol{\mu_P}}{\mathbf{2}} \boldsymbol{\mathcal{P}^2} + \boldsymbol{\chi_{\zeta \in [0,1]}} + \boldsymbol{\chi_{\Gamma \subset D}} + \boldsymbol{l_{TV}} \frac{\mathbf{1}}{|\boldsymbol{\Gamma}|} \int_{\boldsymbol{\Gamma}} |\boldsymbol{d_s \zeta}| \boldsymbol{ds}$$

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Steady model – total variation penalization

Controlling the power jumps ➡ Denoising problem [a]

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Proximal gradient algorithm

$$\mathcal{L}_{ALM}(\Gamma,\zeta,l_C,l_P;\mu_C,\mu_P) = \underbrace{|\Gamma| + l_C C + \frac{\mu_C}{2}C^2 + l_P\mathcal{P} + \frac{\mu_P}{2}\mathcal{P}^2}_{f_{Diff}} \underbrace{+ \chi_{\zeta\in[0,1]} + \chi_{\Gamma\subset D} + l_{TV}\frac{1}{|\Gamma|}\int_\Gamma |d_s\zeta|ds}_{f_{Non\ Diff}}$$

f_{Diff}

Differentiable function :
classic gradient algorithm

$f_{Non\ Diff}$

Non differentiable function but easy computation of the proximal :
proximal algorithm

(a) A. Chambolle, V. Caselles, D. Cremers, M. Novaga, and T. Pock, An introduction to total variation for image analysis, Theoretical foundations and numerical methods for sparse recovery, 9 (2010), p. 227.

Proximal definition [a]

Proximal $$prox_{hf}(y) = argmin_x \left(f(x) + \frac{1}{2h} \|x - y\|\hat{}2 \right)$$

(a) N. Parikh and S. Boyd, Proximal algorithms, Foundations and Trends in optimization, 1 (2013), pp. 123–231.

Proximal definition [a]

Proximal $prox_{hf}(y) = argmin_x \left(f(x) + \frac{1}{2h} \|x - y\|\hat{}2 \right)$

Consists in decreasing the objective function f while remaining close to y

$$prox_{\lambda f}(y) = (I_d + h\partial f)^{-1}$$

$\approx x - \mathrm{h}\nabla f(x)$ If f is differentiable

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Euler explicit scheme $\frac{x^{k+1} - x^k}{h} = -\nabla f(x^k) \Rightarrow x^{k+1} = x^k - h\nabla f(x^k)$

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(a) N. Parikh and S. Boyd, Proximal algorithms, Foundations and Trends in optimization, 1 (2013), pp. 123–231.

Steady model – total variation penalization

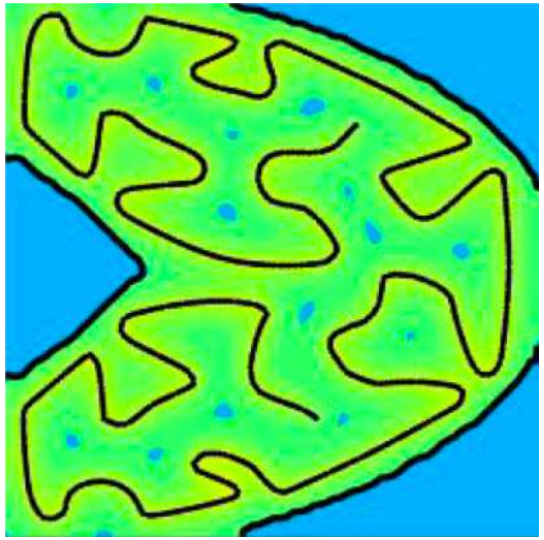
Algorithm

Initialize (Γ^0, ζ^0) and l_ϕ^0, l_M^0, λ_ζ^0
Compute the objective function
Compute the gradient with respect to the path $\nabla_\Gamma \mathcal{L}_\mathrm{ALM}(\Gamma^0, \zeta^0, l_\phi^0, l_\mathrm{M}^0, \lambda_\zeta^0)$
Compute the gradient with respect to the power variable $\nabla_\zeta \mathcal{L}_\mathrm{ALM}(\Gamma^0, \zeta^0, l_\phi^0, l_\mathrm{M}^0, \lambda_\zeta^0)$
while *the stopping criterion is not reached* **do**
    Compute the update steps s_Γ^n and s_ζ^n
    Compute $\tilde{\Gamma}^{n+1} = \Gamma^n - s_\Gamma^n \nabla_\Gamma f_\mathrm{D}(\Gamma^n, \zeta^n, l_\phi^n, l_\mathrm{M}^n, \lambda_\zeta^n)$
    Compute $\tilde{\zeta}^{n+1} = \zeta^n - s_\zeta^n \nabla_\zeta f_\mathrm{D}(\Gamma^0, \zeta^n, l_\phi^n, l_\mathrm{M}^n, \lambda_\zeta^n)$
    Path projection $\Gamma^{n+1} = \mathbb{P}_D(\Gamma^n)$
    Application of the proximal related to ζ: $\zeta^{n+1} = \mathbb{P}_{\zeta \in [0,1]} \left(\mathrm{prox}_{s_\zeta^n l_\mathrm{TV} TV(\tilde{\zeta}^n)} \right)$
    Compute the new objective function
    if *the new objective function is smaller than before (up to a tolerance)* **then**
        Iteration accepted
        Update of the Lagrange multipliers and recompute the objective function
        Compute the shape and power derivatives
        Increase the step coefficients (line search)
    end
    else
        Reject the iteration
        Decrease the coefficients: line only for a line iteration, power only for a power iteration and both if both
    end
end

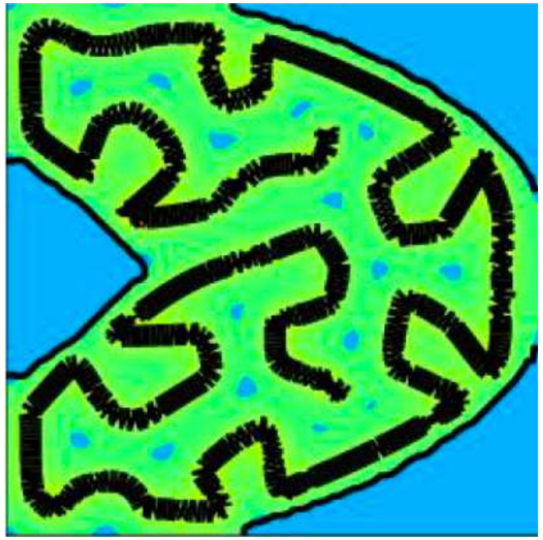
A. Barbero and S. Sra, Fast newton-type methods for total variation regularization, in Proceedings of the 28th International Conference on Machine Learning (ICML-11), Citeseer, 2011, pp. 313–320.

Steady model – total variation penalization

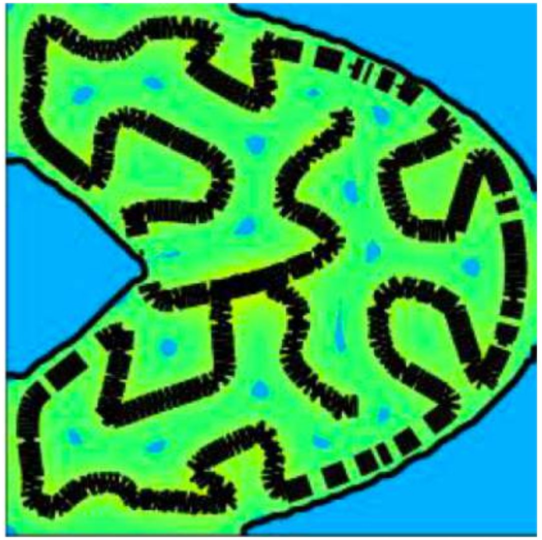
$\zeta_{ini} = 0.5$



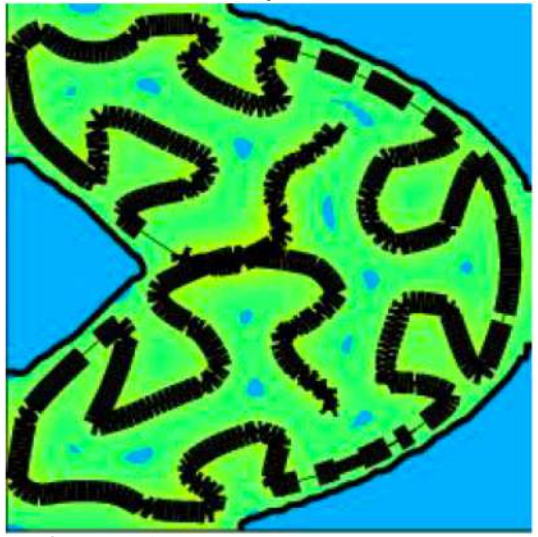
Path only optimization



Path and power optimization
Relaxed problem



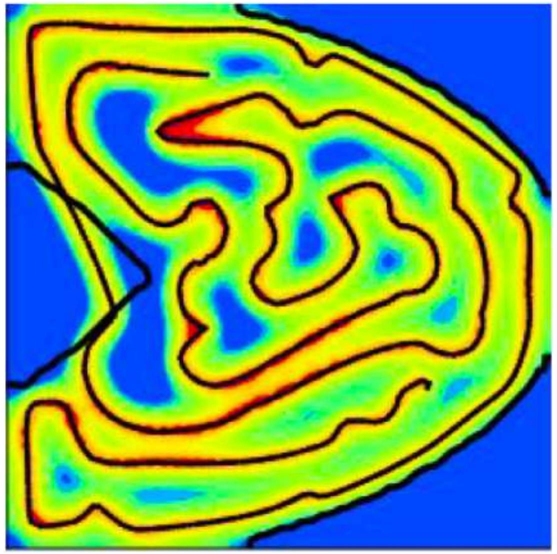
Path and power optimization
Relaxed-penalized problem



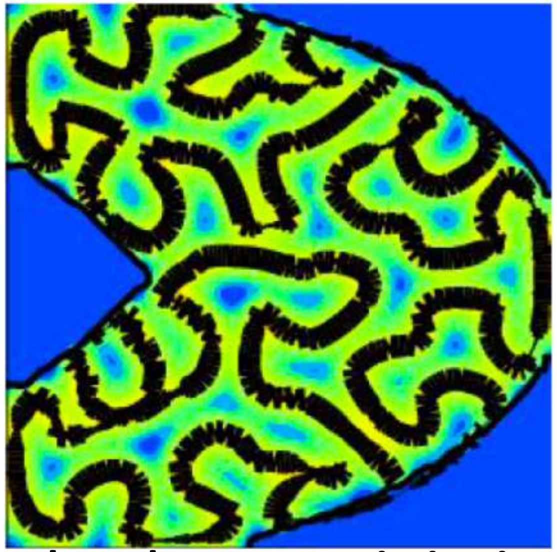
Path and power optimization
Relaxed-penalized and total variation penalization problem

Steady model – total variation penalization

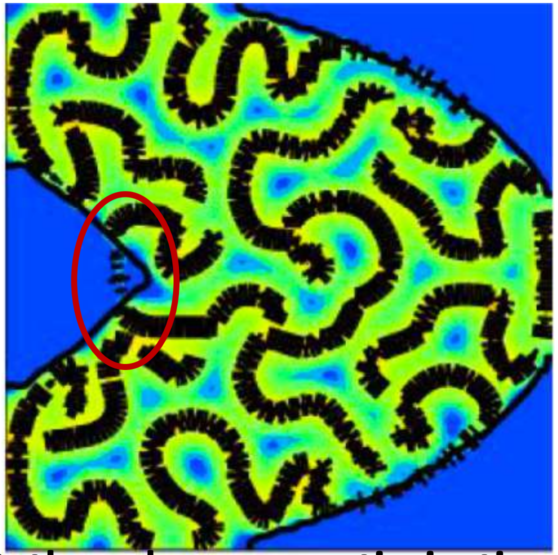
$\zeta_{ini} = 0.5$



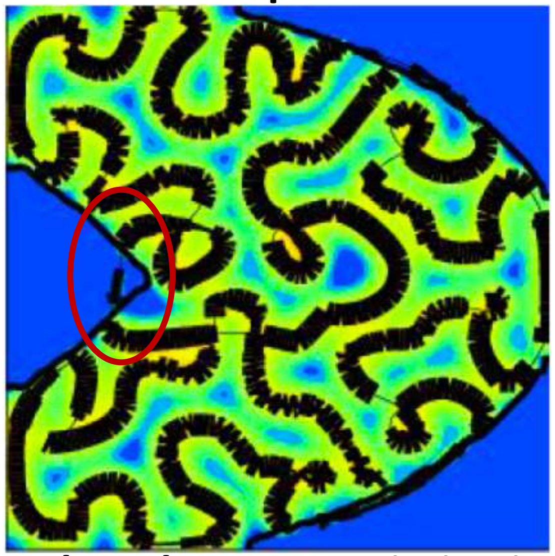
Path only optimization



Path and power optimization
Relaxed problem



Path and power optimization
Relaxed-penalized problem



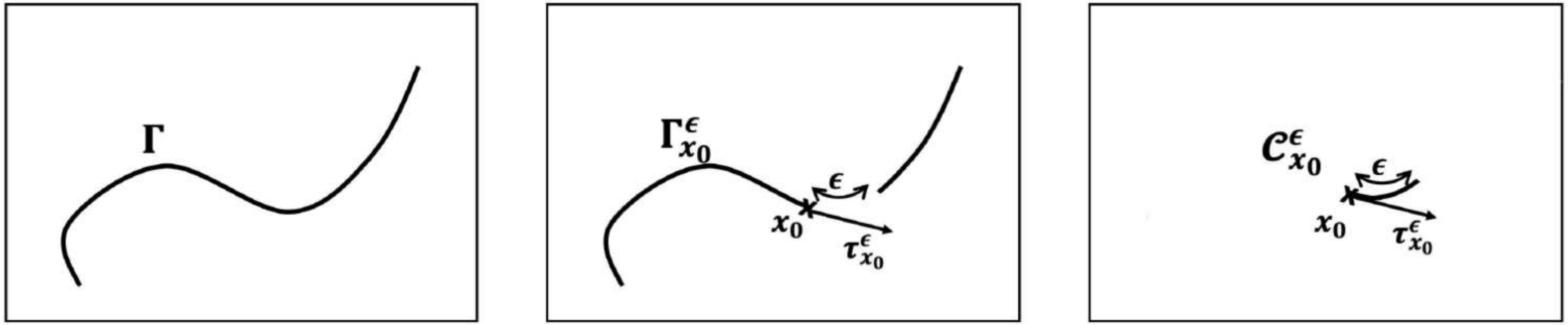
Path and power optimization
Relaxed-penalized and total variation penalization problem

Overview

- Modelling assumptions
- Scanning path optimization
- Modifying the path topology
    - Physical approach: coupling scanning path and power optimization
    - Topology approach: topological optimization of the scanning path

Steady model – topological optimization of the path [a]

Removing part of the path



(a) Path Γ (b) Perturbed path $\Gamma^{\epsilon}_{x_0}$ (2 connected components) (c) Path $\mathcal{C}^{\epsilon}_{x_0}$

$$J(\Gamma^{\epsilon}_{x_0}) = J(\Gamma) - \big(1 - Pp(x_0)\big)\epsilon + o(\epsilon)$$

$$1 - Pp(x_0) > 0 \Rightarrow p(x_0) < \frac{1}{P}$$

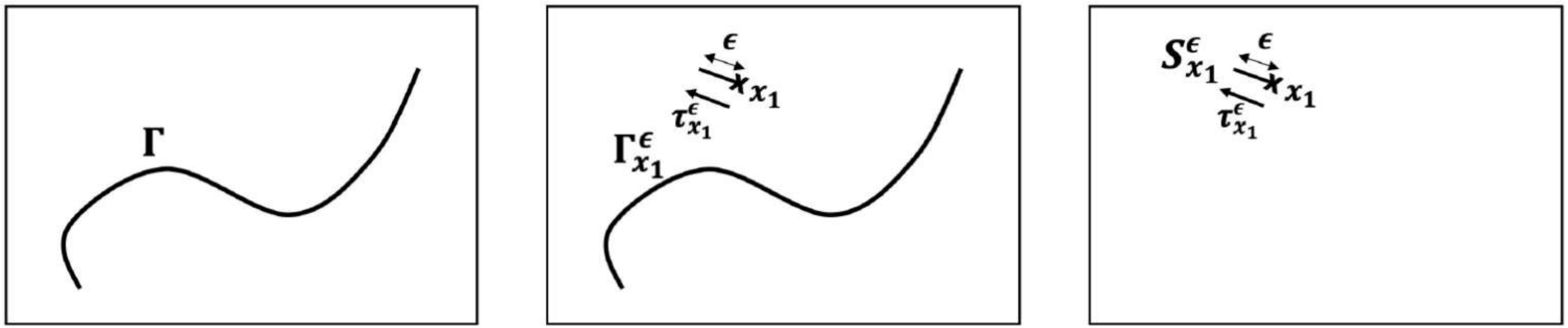
Adjoint function $\boldsymbol{p}$

$$\begin{cases} -\nabla(\lambda\nabla p) + \beta p = 2(l_c + \mu_c C)\left(\left[y_\phi - y\right]^+ - \left[y - y_{M,in}\right]^+ - \left[y - y_{M,out}\right]^+\right), & x \in D \\ \lambda\partial_n y = 0, & x \in \partial D \end{cases}$$

(a) L. Rakotondrainibe, Join topology optimisation of part and support for metal additive manufacturing, PhD thesis, Ecole Polytechnique, 2020.

Steady model – topological optimization of the path [a]

Adding part of the path



(a) Path Γ (b) Perturbed path $\Gamma^{\epsilon}_{x_1}$ (2 connected components) (c) Path $\mathcal{S}^{\epsilon}_{x_1}$

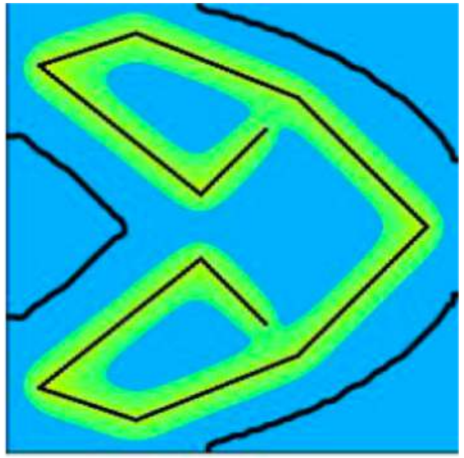
$$J\left(\Gamma^{\epsilon}_{x_1}\right) = J(\Gamma) - \left(1 - Pp(x_1)\right)\epsilon + o(\epsilon) \qquad 1 - Pp(x_1) < 0 \Rightarrow p(x_1) > \frac{1}{P}$$

Adjoint function p

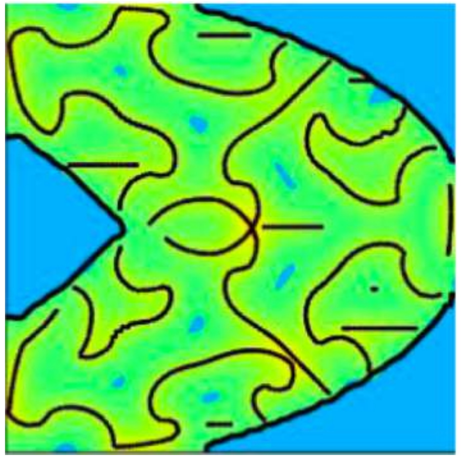
$$\begin{cases} -\nabla(\lambda\nabla p) + \beta p = 2(l_c + \mu_c C)\left(\left[y_\phi - y\right]^+ - \left[y - y_{M,in}\right]^+ - \left[y - y_{M,out}\right]^+\right), & x \in D \\ \lambda\partial_n y = 0, & x \in \partial D \end{cases}$$

(a) L. Rakotondrainibe, Join topology optimisation of part and support for metal additive manufacturing, PhD thesis, Ecole Polytechnique, 2020.

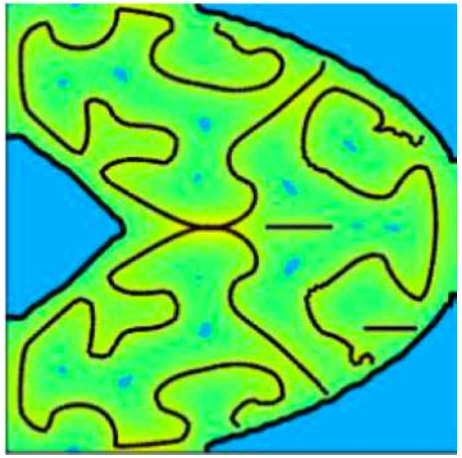
Steady model – Results - aluminum



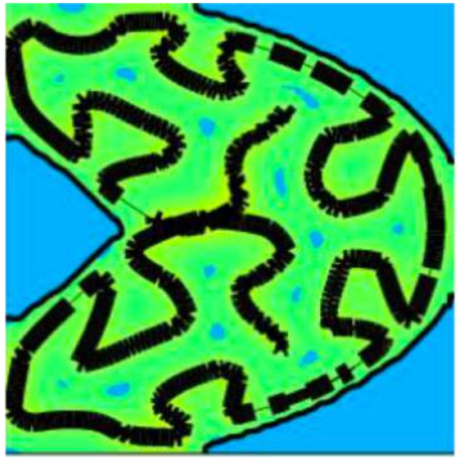
Initialization



Topology optimization
$TV \leq 15$

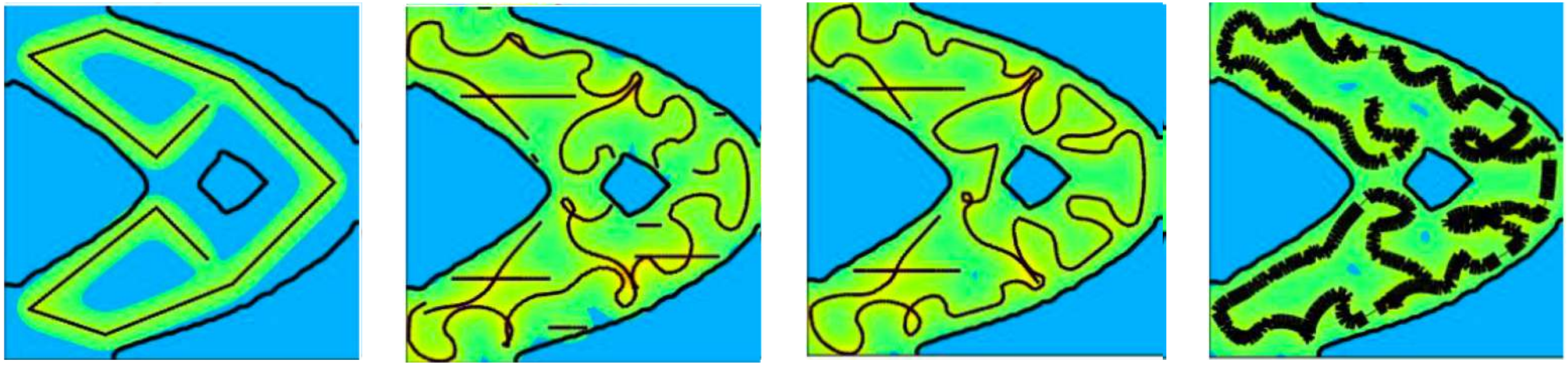


Topology optimization
$TV \leq 5$



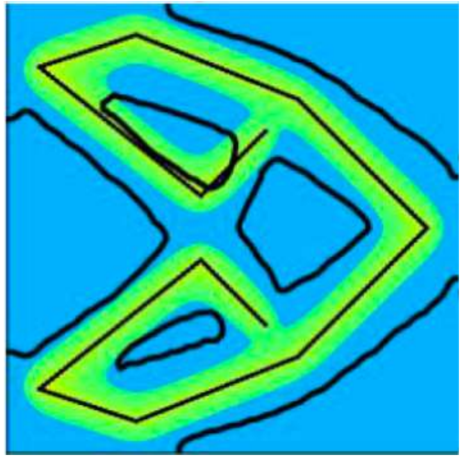
Power optimization

Steady model – Results- aluminum

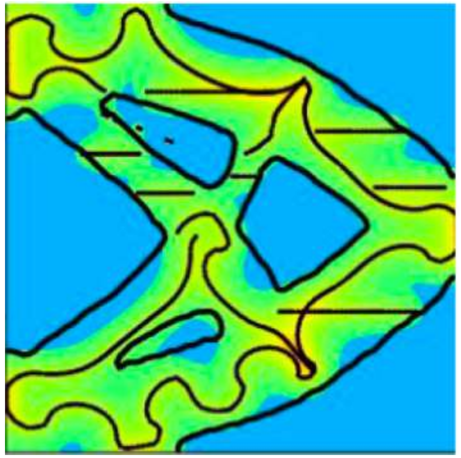


Initialization | Topology optimization $TV \leq 15$ | Topology optimization $TV \leq 5$ | Power optimization

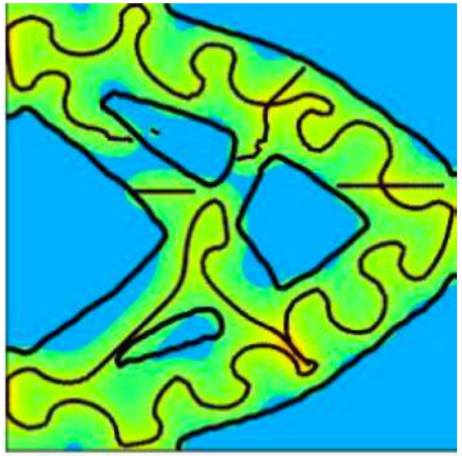
Steady model – Results - aluminum



Initialization



Topology optimization
$TV \leq 15$



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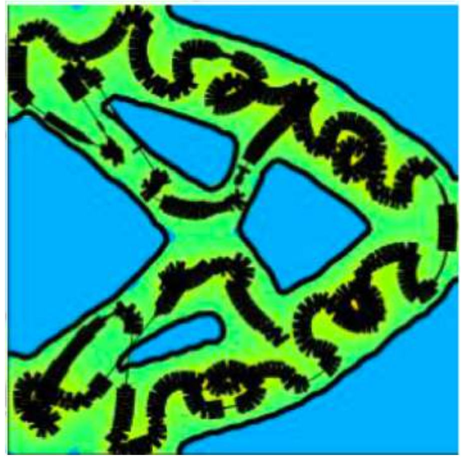
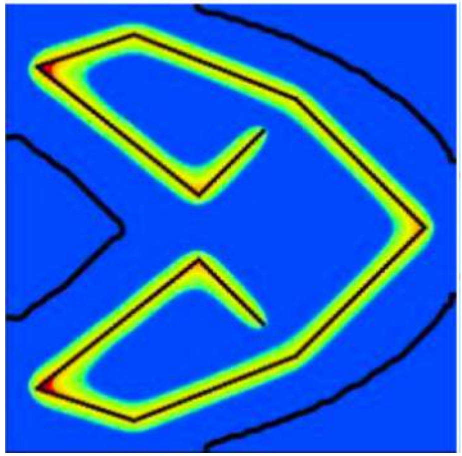
$TV \leq 5$



Power optimization

Steady model – Results- titanium



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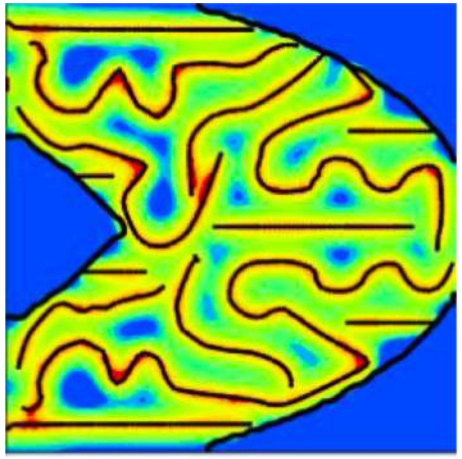

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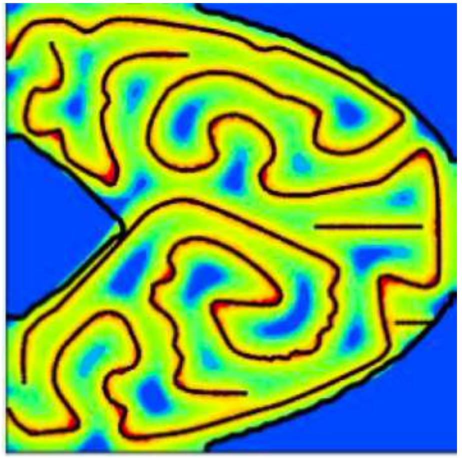
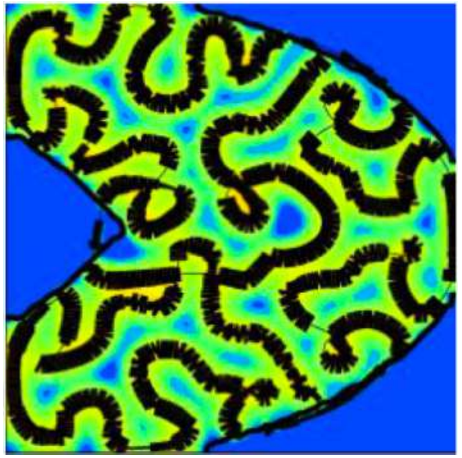
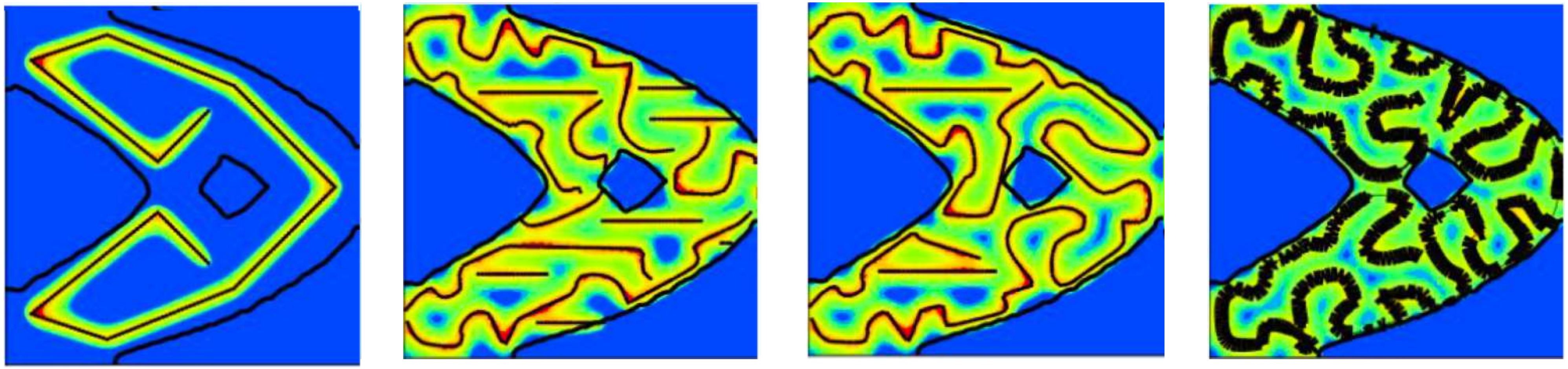


Topology optimization
$TV \leq 5$



Power optimization

Steady model – Results - titanium



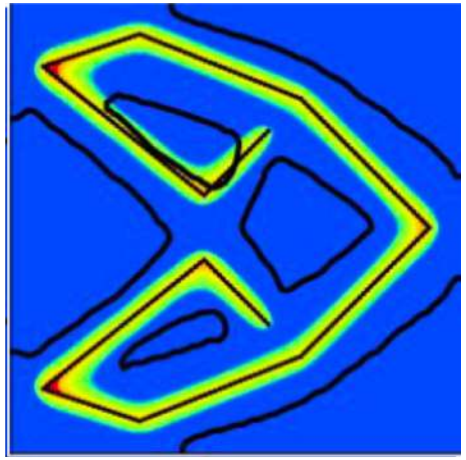
Initialization

Topology optimization
$TV \leq 15$

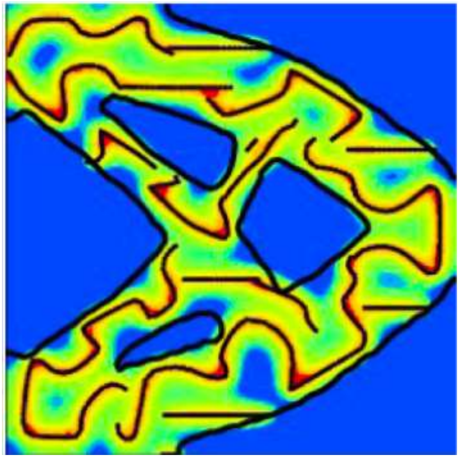
Topology optimization
$TV \leq 5$

Power optimization

Steady model – Results- titanium



Initialization



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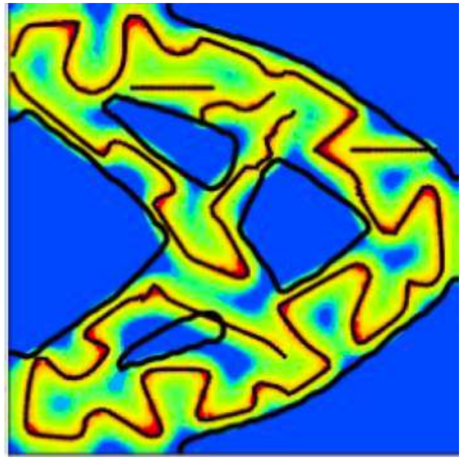

Topology optimization
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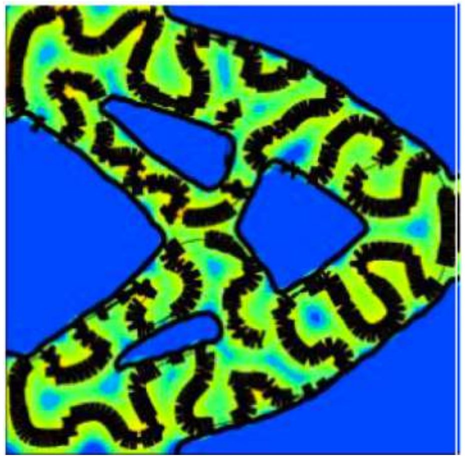

Power optimization

Conclusions

Power based strategy

“Physical” representation of the path connected components :

- Continuous consideration of the number of connected components
- Number of connected components limited by an optimization constraint

Reasonable computational costs

Topology optimization

“Artificial” representation of the path connected components :

- Discrete consideration of the number of connected components
- Number of connected components limited by the algorithm settings

High computational costs

Conclusions and perspectives

Adding the path topology to the optimization

- Confirms the importance of allowing topology modifications
- Remains a complex feature to control

Perspectives

- Improve the topology optimization
- Adapt the constraint to reality: advantage the phase constraint, define “steady state” constraints modelling transient quantities (kinematics) to take benefit from the very easy resolution process and shape optimization theory
- Optimize in the transient (general) model
- Add the resolution of a mechanical problem (full resolution or inherent strain method)
- 3D considerations
- Concurrent optimization of the scanning path and of the shape of the part to build

References

References on which this talk is based on

- M.Boissier, G.Allaire, C.Tournier, *Additive Manufacturing Scanning Paths Optimization Using Shape Optimization Tools*, SMO, 61:6, pp. 2437-2466 (2020)

- M.Boissier, G.Allaire, C.Tournier, *Concurrent shape optimization of the part and scanning path for additive manufacturing*, submitted (2021). ⟨hal-03124075⟩.

- M.Boissier, Coupling structural optimization and trajectory optimization methods in additive manufacturing, PhD thesis (2020)

Further reference for path optimization considering unsteadiness :

- M.Boissier, G.Allaire, C.Tournier, *Time dependent scanning path optimization for the powder bed fusion additive manufacturing process*, submitted (2021). ⟨hal-03202102⟩.

Thank you