DE LA RECHERCHE À L'INDUSTRIE

EXTENSION OF A ROE TYPE SCHEME WITH LOW MACH CORRECTION TO THE HRM TWO-PHASE FLOW MODEL

SMAI 2021 - Méthodes numériques pour la simulation d'écoulements à bas nombre de Mach

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Context and motivation

The HRM model and numerical approximation

Extension of the new Roe scheme

Some steady test case in 1D and 2D

- Boiling 1D-channel with a singular pressure loss
- Flow rate balancing with 3 singular pressure losses
- Incoming flow in two channels with thermal power

Conclusion and perspective

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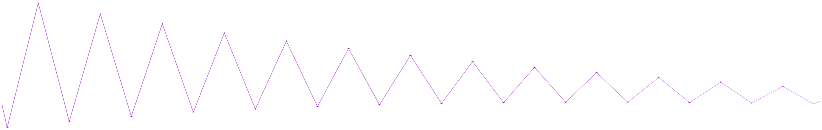
FLICA4 : a three-dimensional two-phase flow computer code with advanced numerical methods for nuclear applications. *I. Toumi, A. Bergeron, D. Gallo, E. Royer and D. Caruge. Nuclear Engineering and Design, vol. 200, p. 139-155, 2000.*

- Three-dimensional thermal-hydraulic code for simulating steady-state and transient two-phase flows in **the core of a PWR**.
- System of equations based on the **Drift-flux** mixture model with liquid-vapor water EOS.
- Colocated finite volume method with **VF-Roe** scheme for convective fluxes.
- Linearized time implicit formulation solved with a Newton method.
- Many situations are at **low Mach number** $M = \frac{u}{a} \ll 1$ (≈ 0.001 to 0.01) and meshes are **cartesian** (2D cartesian extruded grids).

Analysis of Godunov Type Schemes Applied to the Compressible Euler System at Low Mach Number. *S. Dellacherie, JCP, vol. 229, p. 978-1016, 2010.*

- *Observation :* Godunov type schemes are inaccurate at Low Mach number on cartesian grids.
- *Cause :* wrong order of the numerical diffusion in the pressure gradient.
- *Solution : « pressure correction »* by changing the order of the guilty term.
- *Undesirable effect :* **checkerboard modes** (on $(p, \mathbf{u})$ profiles).



Those can be **amplified by stiff or discontinuous source terms**. Then they disturb convergence to steady-state or even lead to code failure...

A low Mach correction able to deal with low Mach acoustics. *P. Brunel, S. Delmas, J. Jung, and V. Perrier, JCP, vol. 378, p. 723-759, 2019.*

- A new Roe scheme for low Mach number flows which :
  1. is **accurate** at low Mach number ;
  2. is **stable** with same CFL restriction as the standard Roe scheme ;
  3. gives the **right order of precision** for higher order spatial discretization and without any CFL degradation (as for the Dellacherie fix for instance).
- But more importantly for us : it does not show **any checkerboard modes.**
- We want to extend this scheme to our applications : the **HRM two-phase flow mixture model** with liquid-vapor water type EOS.

Plan

We consider the two-phase flows **Homogeneous Relaxation Model** under the following contracted form :

$$\partial_t U + \nabla_{\mathbf{x}} \cdot F(U) = S(U), \quad \mathbf{x} \in \mathbb{R}^3,\ t > 0,$$

where

$$U = \begin{pmatrix} \rho \\ \rho c \\ \rho \mathbf{u} \\ \rho E \end{pmatrix}, \quad F(U) = \begin{pmatrix} \rho \mathbf{u}^T \\ \rho c \mathbf{u}^T \\ \rho \mathbf{u} \otimes \mathbf{u} + p\mathbf{I} \\ \rho E \mathbf{u} + p\mathbf{u}^T \end{pmatrix}, \quad S(U) = \begin{pmatrix} 0 \\ \omega \rho (c_{eq} - c) \\ \rho \mathbf{g} - \frac{1}{2} K(\mathbf{x}) \rho \mathbf{u} |\mathbf{u}| \\ Q(\mathbf{x}, t) \end{pmatrix}.$$

Conservative variables in vector U are associated to the **mixture** and c is the **vapor mass fraction**.

$$\rho e = \rho E - \rho |\mathbf{u}|^2/2 \quad \text{the internal energy of the mixture,}$$

$$c_{eq} = (h - h_{ls}(p))/\mathcal{L} \quad \text{the vapor mass fraction at equilibrium,}$$

$$p = p(\rho, \rho c, \rho e) \quad \text{the mixture pressure law,}$$

$$a^2 = \frac{\partial p}{\partial \rho} + c\frac{\partial p}{\partial (\rho c)} + h\frac{\partial p}{\partial (\rho e)} \quad \text{the square mixture sound velocity.}$$

We define the following scaling variables :

$$\bar\rho = \frac{\rho}{\rho_0}, \qquad \bar c = c \qquad \bar p = \frac{p}{p_0}, \qquad \bar{\mathbf{u}} = \frac{\mathbf{u}}{u_0}, \qquad \bar E = \frac{E}{a_0^2}$$

$$\bar{\mathbf{x}} = \frac{\mathbf{x}}{l_0}, \qquad \bar t = \frac{t}{t_0}, \qquad t_0 = \frac{l_0}{u_0}.$$

Under condition $p_0 = \rho_0 a_0^2$ the dimensionless HRM system writes :

$$\begin{cases} \partial_{\bar t}\bar\rho + \nabla_{\bar{\mathbf{x}}} \cdot (\bar\rho\bar{\mathbf{u}}) = 0, \\ \partial_{\bar t}(\bar\rho\bar c) + \nabla_{\bar{\mathbf{x}}} \cdot (\bar\rho\bar c\bar{\mathbf{u}}) = C_\omega\bar\omega\bar\rho(\bar c_{eq} - \bar c), \\ \partial_{\bar t}(\bar\rho\bar{\mathbf{u}}) + \nabla_{\bar{\mathbf{x}}} \cdot (\bar\rho\bar{\mathbf{u}} \otimes \bar{\mathbf{u}}) + \frac{1}{M^2}\nabla_{\bar{\mathbf{x}}}\bar p = \frac{1}{Fr}\bar\rho\bar{\mathbf{g}} - C_K\frac{1}{2}\bar K(\bar{\mathbf{x}})\bar\rho\bar{\mathbf{u}}|\bar{\mathbf{u}}|, \\ \partial_{\bar t}(\bar\rho\bar E) + \nabla_{\bar{\mathbf{x}}} \cdot (\bar\rho\bar E\bar{\mathbf{u}} + \bar p\bar{\mathbf{u}}) = C_Q\bar Q(\bar{\mathbf{x}}, \bar t). \end{cases}$$

where $M = \frac{u_0}{a_0}$. We also suppose that $\frac{1}{Fr}, C_K \ll \frac{1}{M}$.

We perform the following asymptotic expansion :

$$\bar{\varphi}\left(\bar{\mathbf{x}}, \bar{t}, M\right) = \bar{\varphi}^{(0)}\left(\bar{\mathbf{x}}, \bar{t}\right) + M\bar{\varphi}^{(1)}\left(\bar{\mathbf{x}}, \bar{t}\right) + M^2\bar{\varphi}^{(2)}\left(\bar{\mathbf{x}}, \bar{t}\right) + \ldots$$

on each non-dimensioned variables $\bar{\varphi} \in \left\{\bar{\rho}, \bar{c}, \bar{\mathbf{u}}, \bar{E}, \bar{p}\right\}$.

- **Orders** M^{-2} and M^{-1} :

$$\nabla \bar{p}^{(0)} = \nabla \bar{p}^{(1)} = 0.$$

- This leads to the non-dimensioned pressure field behaviour

$$\bar{p}(\bar{\mathbf{x}}, \bar{t}, M) = \bar{p}^{(0)}(\bar{t}) + M^2\bar{p}^{(2)}(\bar{\mathbf{x}}, \bar{t}) + \mathcal{O}\left(M^3\right).$$

In other words : at low Mach number, the spatial variation of the non-dimensioned pressure field is of magnitude $\mathcal{O}\left(M^2\right)$. See *Klainerman Majda 1981, Guillart Viozat 1999, Dellacherie 2010...*

- We numerically solve the HRM system with a **finite volume approach** on mesh $\Omega = \left\{\Omega_i \subset \Omega, i = 1, \ldots, N\right\}$ which gives the following semi-discrete formulation

$$\partial_t U_i + \frac{1}{\left|\Omega_i\right|} \sum_{\Gamma_{ij} \subset \overline{\Omega}_i \cap \overline{\Omega}_j} \left|\Gamma_{ij}\right| \Phi_{ij}\left(U_i, U_j, \mathbf{n}_{ij}\right) = S\left(U_i\right), \ \forall \Omega_i \subset \Omega.$$

- The numerical fluxes Φ_{ij} on each face $\Gamma_{ij} \subset \overline{\Omega}_i \cap \overline{\Omega}_j$ are given in FLICA4 by the **VF-Roe scheme** : we solve a linearized Riemann problem between cells Ω_i and Ω_j.

- The **Roe matrix** is used to linearize the jacobian matrix of the convective fluxes in order to solve the Riemann problem.

The numerical fluxes are defined by :

$$\Phi_{ij}\left(U_i, U_j, \mathbf{n}_{ij}\right) = F\left(U^*\left(U_i, U_j, \mathbf{n}_{ij}\right)\right)\mathbf{n}_{ij} = \begin{pmatrix} (\rho u)^* \\ (\rho c)^* u^* \\ (\rho u)^* \mathbf{u}^* + p^* \mathbf{n}_{ij} \\ (\rho E)^* u^* + p^* u^* \end{pmatrix},$$

where

$$\begin{aligned} U^*\left(U_i, U_j, \mathbf{n}_{ij}\right) & \quad \text{the solution of the lin. Riemann problem at face } \Gamma_{ij}, \\ u^* = \mathbf{u}^* \cdot \mathbf{n}_{ij}, & \quad \text{the normal velocity at face } \Gamma_{ij}, \\ p^* = p\left(\rho^*, (\rho c)^*, (\rho e)^*\right), & \quad \text{the pressure at face } \Gamma_{ij}. \end{aligned}$$

A more explicit formula can be obtained for p^* :

$$p^* = \frac{p_i + p_j}{2} - \frac{\rho_{ij} a_{ij}}{2} \Delta_{ij} u,$$

where $\Delta_{ij} u = \left(\mathbf{u}_j - \mathbf{u}_i\right) \cdot \mathbf{n}_{ij}$ and $\varphi_{ij}, \varphi \in \{\rho, a\}$, the Roe average density and the sound mixture velocity (see *Toumi, JCP, 1992,* for more details).

In a non-dimensioned form, this pressure at face writes :

$$\bar{p}^* = \frac{\bar{p}_i + \bar{p}_j}{2} - M \frac{\bar{\rho}_{ij}\bar{a}_{ij}}{2} \Delta_{ij}\bar{u}.$$

The upwind term on the right is of magnitude $\mathcal{O}(M\Delta x)$. This leads to

$$\bar{p}(\bar{\mathbf{x}}, \bar{t}) \approx \bar{p}^0(\bar{t}) + M\bar{p}^1(\bar{\mathbf{x}}, \bar{t}) + M^2\bar{p}^2(\bar{\mathbf{x}}, \bar{t}) + \dots$$

At low Mach number, the VF-Roe scheme creates a **spurious component** $\bar{p}^1$ in the non-dimensioned pressure field.

A way to solve this issue is the **pressure correction** technique.

- **VF-Roe All-Mach** pressure correction *(Dellacherie and al. 2016)*

$$p^*_{AM} = \frac{p_i + p_j}{2} - \theta_{ij} \frac{\rho_{ij} a_{ij}}{2} \Delta_{ij} u$$

with

$$\theta_{ij} = \min\left(1, M_{ij}\right), \quad M_{ij} = \frac{|\mathbf{u}_{ij}|}{a_{ij}}.$$

- **VF-Roe FLICA4** pressure correction

$$p^*_{F4} = \frac{p_i + p_j}{2} - \theta_{ij} \frac{\rho_{ij} a_{ij}}{2} \Delta_{ij} u + \frac{u_{ij}}{2|\mathbf{u}_{ij}|} \Delta_{ij} p.$$

Centering the pressure at face by removing spatial diffusion leads to a loss of stability. Then checkerboard modes appear...

Plan

- We define the two dimensionless times associated to the material waves and the acoustic waves as following :

$$\bar{t} = \frac{t}{l_0/u_0},$$
$$\tau = \frac{t}{l_0/a_0} = \frac{t}{l_0/u_0}\frac{1}{u_0/a_0} = \frac{\bar{t}}{M}.$$

- Then we do the asymptotic expansion with these two dimensionless times :

$$\bar{\varphi}\left(\bar{\mathbf{x}}, \bar{t}, \tau, M\right) = \bar{\varphi}^{(0)}\left(\bar{\mathbf{x}}, \bar{t}, \tau\right) + M\bar{\varphi}^{(1)}\left(\bar{\mathbf{x}}, \bar{t}, \tau\right) + M^2\bar{\varphi}^{(2)}\left(\bar{\mathbf{x}}, \bar{t}, \tau\right) + \ldots$$

- Temporal derivative of variable $\bar{\varphi}$ with this new time dependance is :

$$\partial_{\bar{t}}\bar{\varphi}\left(\bar{\mathbf{x}}, \bar{t}\right) = \partial_{\bar{t}}\bar{\varphi}\left(\bar{\mathbf{x}}, \bar{t}, \tau\right) + \frac{1}{M}\partial_{\tau}\bar{\varphi}\left(\bar{\mathbf{x}}, \bar{t}, \tau\right).$$

- When injecting this asymptotic development into the non-dimensioned Euler equations, we obtain the **first-order wave equation** on $(\bar{p}^{(1)}, (\bar{\rho}\bar{\mathbf{u}})^{(0)})$.

- New set of generic dissipative terms are added to semi-discrete wave equation solved with the standard Roe scheme.
- These new diffusive terms are fixed by ensuring the following properties :
  1. a **semi-discrete energy inequality** ;
  2. the **accuracy at low Mach number** by using the fix of *Dellacherie and al.* ;
  3. a **standard CFL** stability condition thanks to a Von Neumann analysis.
- In the following, the new scheme is named **VF-Roe LMAAP** for *Low Mach Acoustic Accuracy Preserving*.

The methodology is applied to the HRM set of equations with a **perfect gas EOS** :

$$p = (\gamma - 1)\rho e.$$

This leads to

$$\Phi_{ij}^{LMAAP}\left(U_i, U_j, \mathbf{n}_{ij}\right) = \begin{pmatrix} (\rho u)^* \\ (\rho c)^* u^* \\ (\rho u)^* \mathbf{u}^* + p^*_{AM} \mathbf{n}_{ij} \\ (\rho E)^* u^* + p^*_{AM} u^* \end{pmatrix} - (1 - \theta_{ij}) \begin{pmatrix} 0 \\ 0 \\ \mp \frac{1}{2\sqrt{d}} \Delta_{ij} p \mathbf{1}_d \\ \frac{H_{ij}}{2a_{ij}} \Delta_{ij} p \pm \frac{\rho_{ij} H_{ij}}{2\sqrt{d}} \Delta_{ij} \mathbf{u} \cdot \mathbf{1}_d \end{pmatrix}.$$

The new terms on the right depend on the problem dimension where d is the dimension and $\mathbf{1}_d = (1, \ldots, 1)^T$.

However, perfect gaz EOS is not an accurate representation of liquid water behaviour. A better approximation would be the **stiffened gaz EOS** :

$$p = (\gamma - 1)\rho(e - q) - \gamma\pi.$$

The same analysis with the SG EOS gives :

$$\Phi_{ij}^{LMAAP}\left(U_i, U_j, \mathbf{n}_{ij}\right) = \begin{pmatrix} (\rho u)^* \\ (\rho c)^* u^* \\ (\rho u)^* \mathbf{u}^* + p^*_{AM} \mathbf{n}_{ij} \\ (\rho E)^* u^* + p^*_{AM} u^* \end{pmatrix} - (1 - \theta_{ij}) \begin{pmatrix} \frac{1}{2a_{ij}} \Delta_{ij} p \pm \frac{\rho_{ij}}{2\sqrt{d}} \Delta_{ij} \mathbf{u} \cdot \mathbf{1}_d \\ 0 \\ \mp \frac{1}{2\sqrt{d}} \Delta_{ij} p \mathbf{1}_d \\ \frac{H_{ij}}{2a_{ij}} \Delta_{ij} p \pm \frac{\rho_{ij} H_{ij}}{2\sqrt{d}} \Delta_{ij} \mathbf{u} \cdot \mathbf{1}_d \end{pmatrix}.$$

The main difference with PG EOS is that the **mass flux is also corrected**. The new right part of the fluxes is not applied for the wall boundary type conditions.

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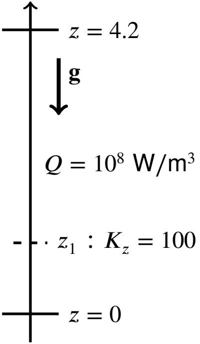
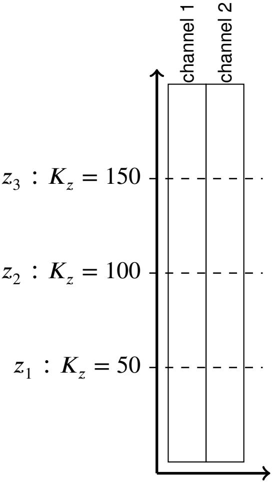
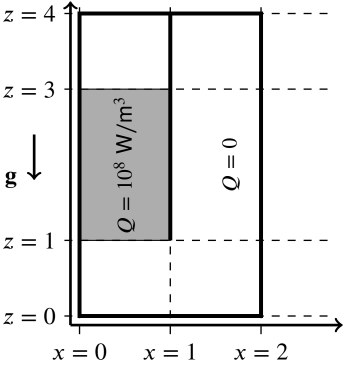


Boiling 1D-channel with a singular pressure loss



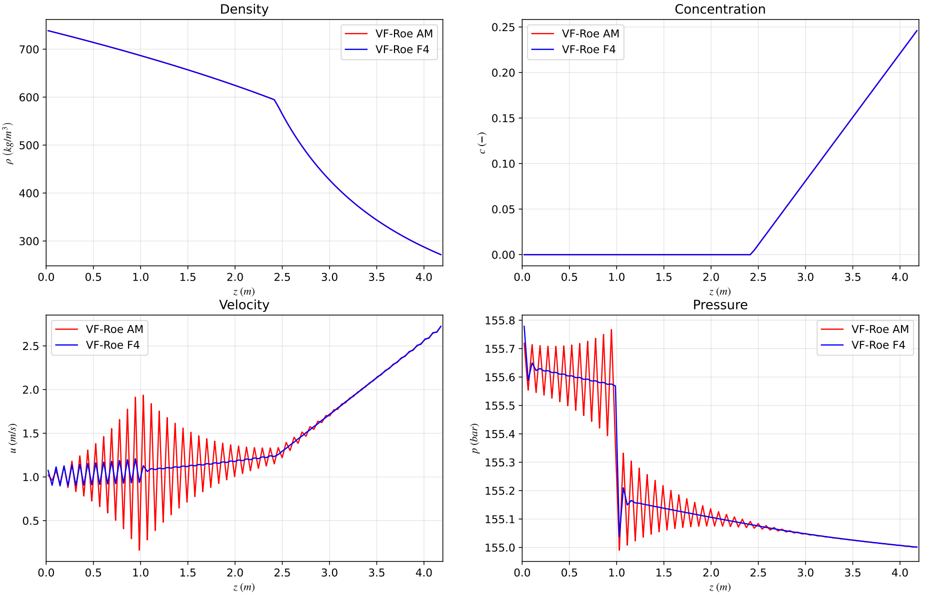
Flow rate balancing with 3 singular pressure losses



Incoming flow in two channels with thermal power

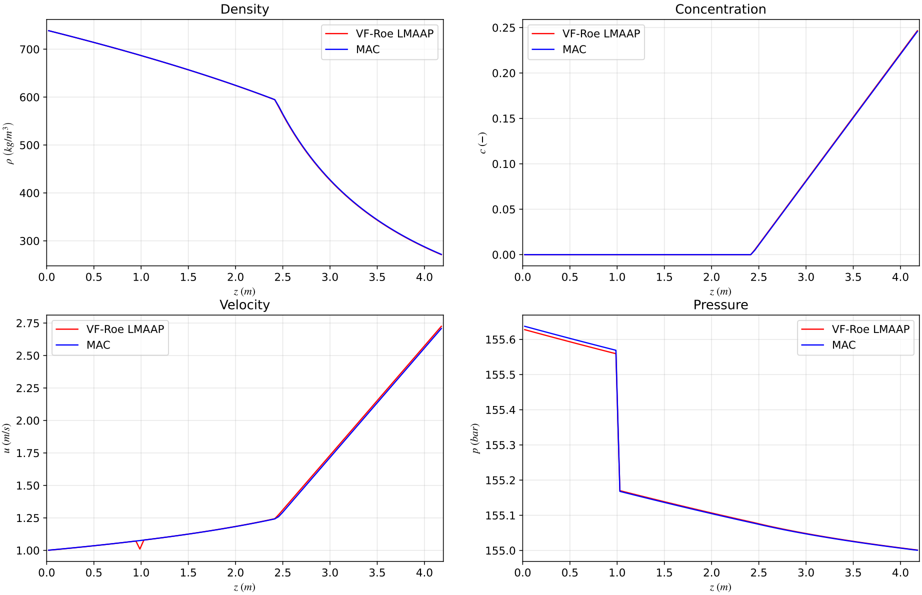
Boiling 1D-channel with a singular pressure loss

VF-Roe AM vs VF-Roe F4



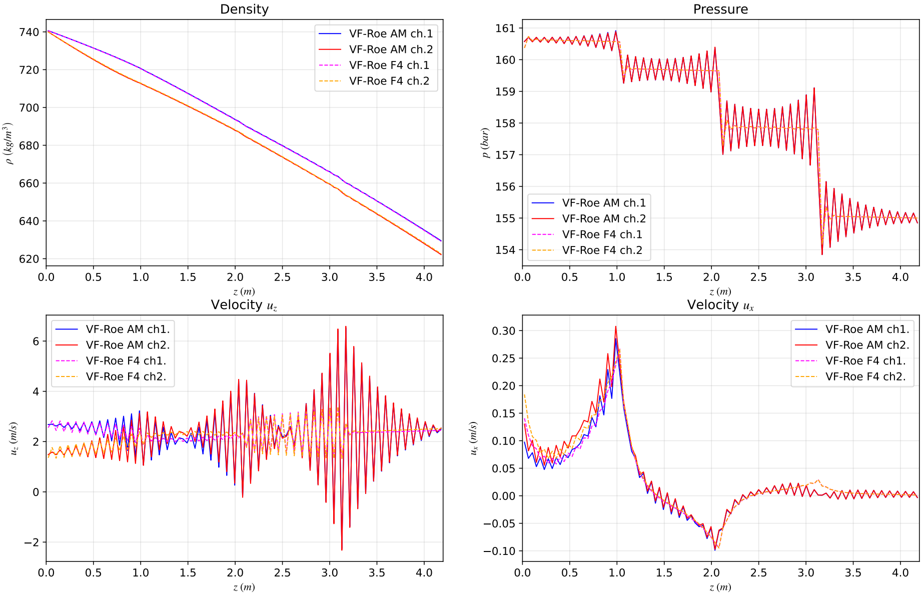
Boiling 1D-channel with a singular pressure loss

VF-Roe LMAAP vs MAC



Flow rate balancing with 3 singular pressure losses

VF-Roe AM vs VF-Roe F4



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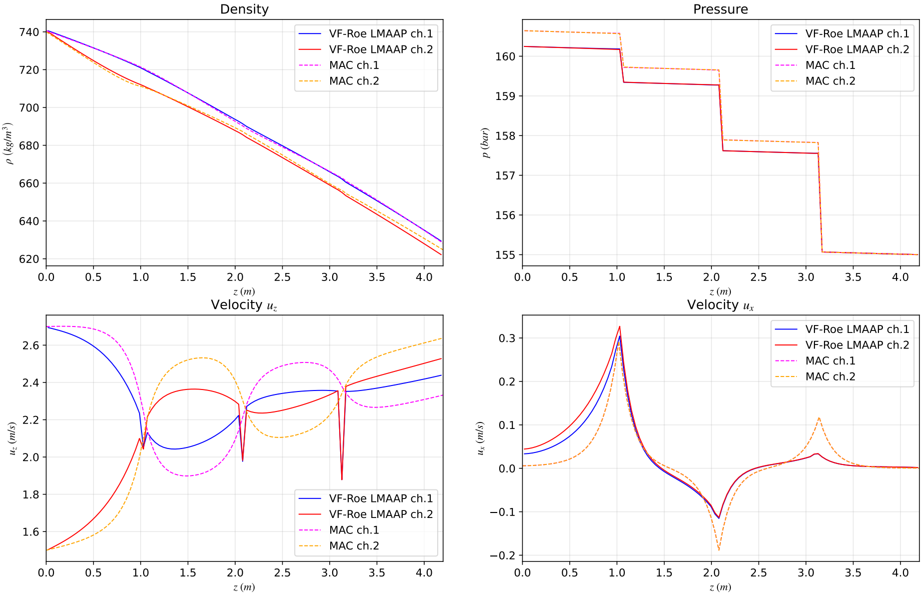
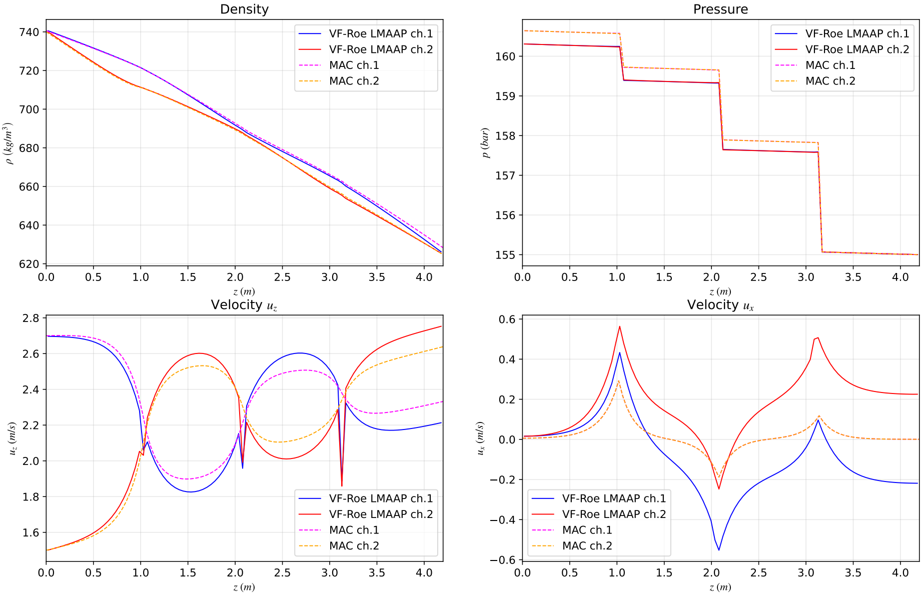


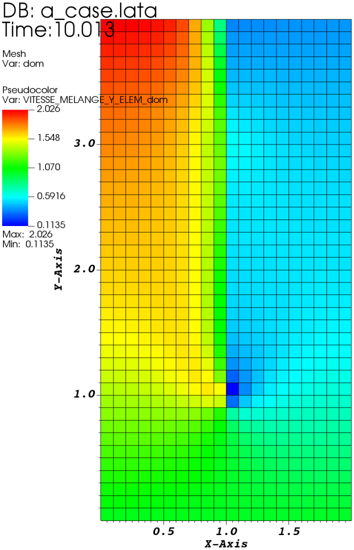
Flow rate balancing with 3 singular pressure losses

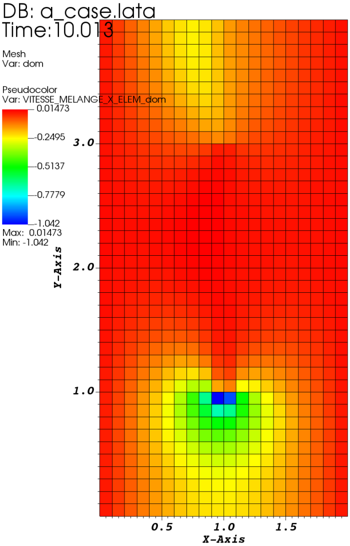
VF-Roe LMAAP ($\theta_{ij} = 0$) vs MAC



Incoming flow in two channels with thermal power

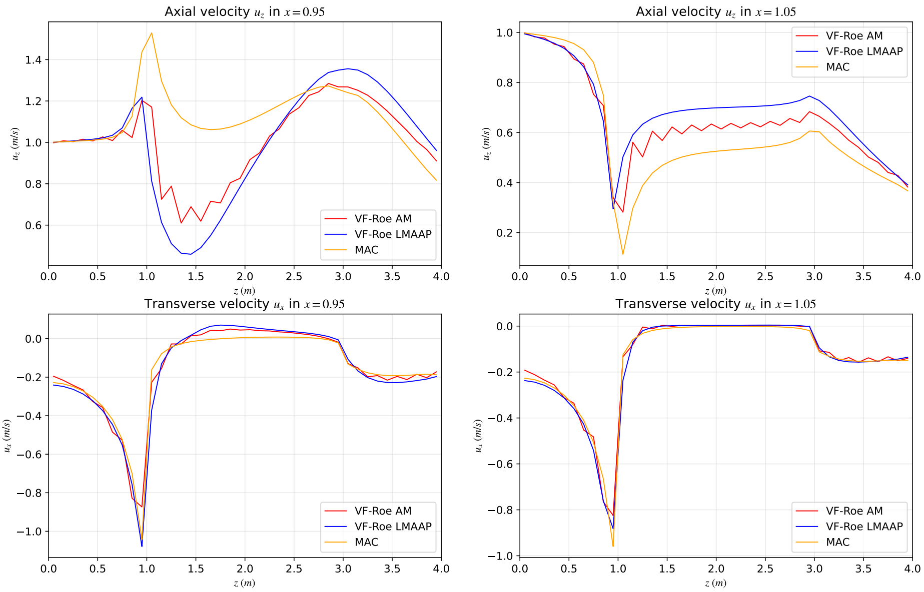
Results for u_z and u_x (MAC scheme)





Incoming flow in two channels with thermal power

VF-Roe AM vs VF-Roe LMAAP vs MAC



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- As promised, the new LMAAP scheme **does not show any checkerboard modes**, even with « mean » source terms (singular pressure loss).
- However, we still have **issues** when applying the new corrected fluxes in **all directions** for $d > 1$.
- All tests work fine when applying the LMAAP fluxes only in the axial direction with $d = 1$ (main flow direction) and the Dellacherie fix only in the transverse direction : there are no checkerboard modes and we obtain the right physical solution.

- Potential programming bug ? Effect of asymetrical flows ? Influence of walls BC ?
- Further investigations are required...
- Also we would have to develop a LMAAP scheme for HRM model with the mixture EOS of the general form :

$$p = p(\rho, \rho c, \rho e).$$

Thank you for your attention