Navigation of Flagellated Micro-Swimmers

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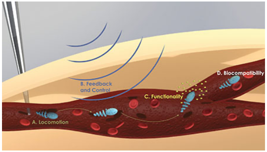
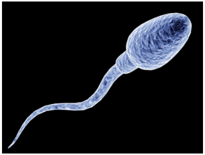
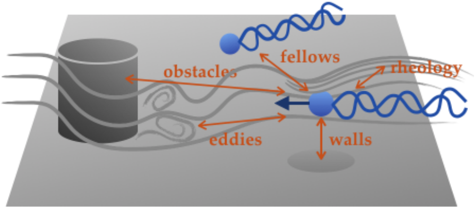
Challenges of micro-swimming



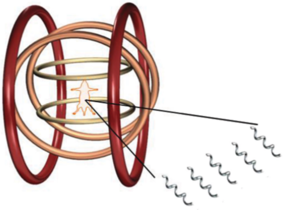
- Medical applications : driving micro-robots through our body
- Multidisciplinary fields (Maths, Physics , Biology and Robotic)
- Biomimetism



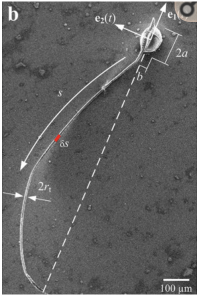


Flagellated locomotion

- Breakthrough → in vivo experiments
- Problems : rigid robots
- The hope of the flagellated locomotion ?
  - Capability to adapt the strategy
- How to control a flagellated robot ?
  - Mathematical modeling
  - Control - optimization tools



[B. Nelson et al., 2015]



[I. S. M. Khalil et al. 2019]

Content

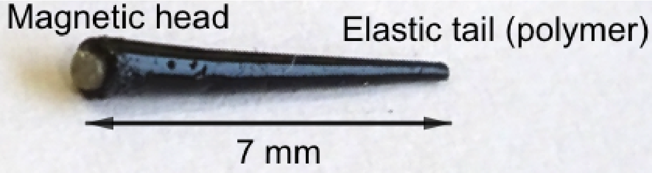
1. Optimization/Control of the magnetic micro-robot
   with Y. El Alaoui Faris, J.-B. Pomet, S. Régnier

2. Accurate simulations for articulated/flagellated swimmer
   with L. Berti, V. Chabannes, C. Prud'Homme

3. Smart swimmer in a turbulent flow
   with J. Bec, R. Chesneaux

Artificial flexible magnetic micro-swimmers

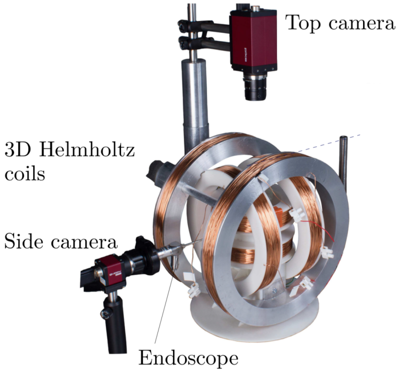
- Imitate a sperm cell propulsion
- Flexible tail + magnetized head
- Magnetic field for producing a flagella-beat waveform
- Length $\sim$ 7mm - Thickness $\sim$ 1.5mm
  $\rightarrow$ Highly viscous fluid
  $\rightarrow$ Low Reynolds Number



Experimental swimmer, ISIR, Sorbonne university, Paris (S. Régnier)

Experimental Setup - Magnetic Field Generation

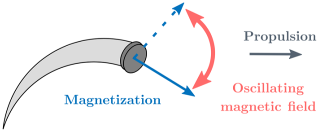
- 3 orthogonal Helmholtz coils.
- 2 cameras.

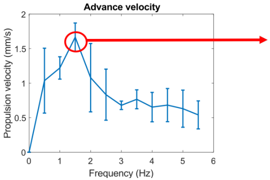


Controlling using a sinusoidal field

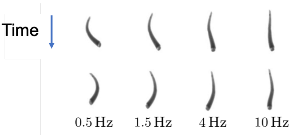
$$B_{\parallel}\mathbf{x}_{\parallel} + B_{\perp}\sin(2\pi f t)\mathbf{x}_{\perp}$$

- Planar displacement ($\mathbf{x}_{\parallel}$-direction)
- Frequency (f) $\Rightarrow$ deformation $\Rightarrow$ displacement





Optimal frequency

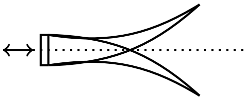


[A. Oulmas, N. Andreff, S. Régnier, 2016]

Link

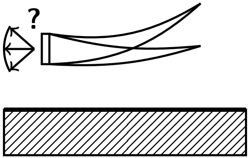
Challenges

- Symmetric field produces a symmetric deformation
- $\Rightarrow$ a transversal deplacement



Sinusoidal field to control the flexible swimmer

- is not optimized
- does not work in the case of complex geometry / bodily fluid



Content

Question

How to find another magnetic field to drive a magnetic flexible micro-swimmer ?

Answer

Optimization problem

Difficulties for predictions

- Hydrodynamic : interaction between fluid and swimmer
- Elasticity : tail's deformation deriving from the magnetic field

Full Mathematical Model

- Fluid equations

$$\begin{cases} \rho(\partial_t u + u \cdot \nabla u) - \mu \Delta u + \nabla p = 0, & \text{in } \mathcal{F}_t, \\ \nabla \cdot u = 0, & \text{in } \mathcal{F}_t, \\ u = U(t), & \text{on } \partial \mathcal{N}_t \ , \\ m\dot{\mathbf{v}} = -F_{fluid} \ , & \\ J\dot{\Omega} = -M_{fluid} \ , & \end{cases}$$

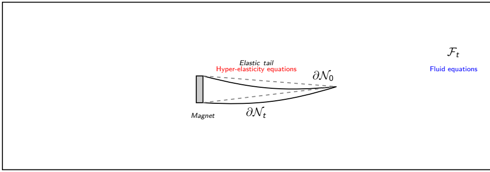
where $U := \mathbf{v} + \Omega \times (x - x^{CM}(t)) + u_d(t)$

- Hyper-elasticity equations

$$\left\{ \rho_s \frac{\partial^2 \eta}{\partial t^2} - \nabla_X \cdot (F\Sigma) = F_{prop} \ , \qquad \text{on } \mathcal{N}_0. \right.$$

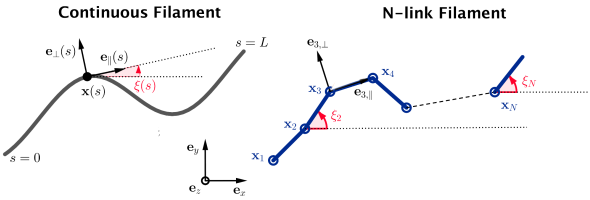
- Coupling

$$\begin{cases} \dfrac{\partial \eta}{\partial t} = u_d(t, x), & \text{on } \partial \mathcal{N}_0, \\ F\Sigma n = \sigma_f n, & \text{on } \partial \mathcal{N}_0. \end{cases}$$



Modeling : main ingredients

- Discretization of the shape
  - 2D N-link swimmer [Alouges, DeSimone, Giraldi, Zoppello, 2013]
  - 3D N-link swimmer [Y. El Alaoui-Faris, J.B. Pomet, S. Régnier, L. Giraldi, 2020]



- Hydrodynamics (h) : Resistive Force Theory

- Elasticity (el) : spring at the junctions

- Magnetism (m) : torques which tend to align the magnet

Dynamics

The dynamics of the micro-robot is governed by an ODE linear with respect to the external magnetic field (i.e., control function **B**) with a drift term,

$$\begin{pmatrix} \dot{\mathbf{X}} \\ \dot{\mathbf{\Theta}} \\ \dot{\mathbf{\Phi}} \end{pmatrix} = \underbrace{\mathbf{F}_0(\mathbf{\Theta}, \mathbf{\Phi})}_{\text{Restoring force}} + \mathbf{F}_1(\mathbf{\Theta}, \mathbf{\Phi})\mathbf{B}(t)\,,$$

Avantage

- Easy to assemble and to compute

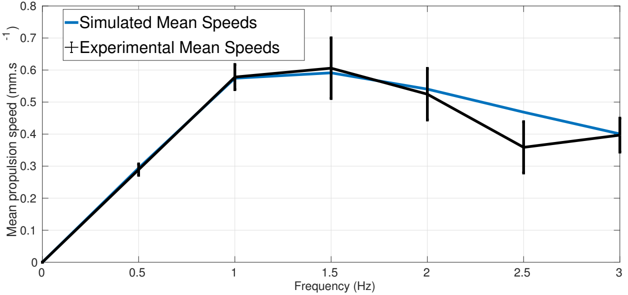
Inconvenient

- Lose complexity of surrounding fluid environnement

RFT in good agreement

- Goal : match the propulsion characteristics

  $\rightarrow$ Velocity-Frequency response to a sinusoidal magnetic field
  $\rightarrow$ Identification of elastic and hydrodynamical parameters



Optimal control problem

- for a given T, find **B** in such a way that the mean speed of the swimmer is maximized for a x-displacement

$$\begin{cases} \min -\dfrac{x(T)}{T} \\ \begin{bmatrix} \dot{\boldsymbol{X}} \\ \dot{\boldsymbol{\Theta}} \\ \dot{\boldsymbol{\Phi}} \end{bmatrix} = \mathbf{F}(\boldsymbol{\Theta}, \boldsymbol{\Phi}, \boldsymbol{B}) \\ \mathbf{B} \text{ bounded} \\ \text{orientation and shape T-periodic} \leftrightarrow \text{stroke constraint} \end{cases}$$

- With this framework, there are many approaches

Several approaches

- Using Pontryaguine principle (or equivalently Lagrange equations)
  [Alouges, Difratta, 2019]
  [Alouges, DeSimone, Giraldi, Or, Weizel, 2019]
  [Loheac and al., 2012]

- Using a direct method
  [Giraldi, Martinon, Zoppello, 2015]
  [Alouges, DeSimone, Heltai, 2013]

- Performing the optimization on specific family of functions
  [Tam, Hosoi, 2007]

Difficulties

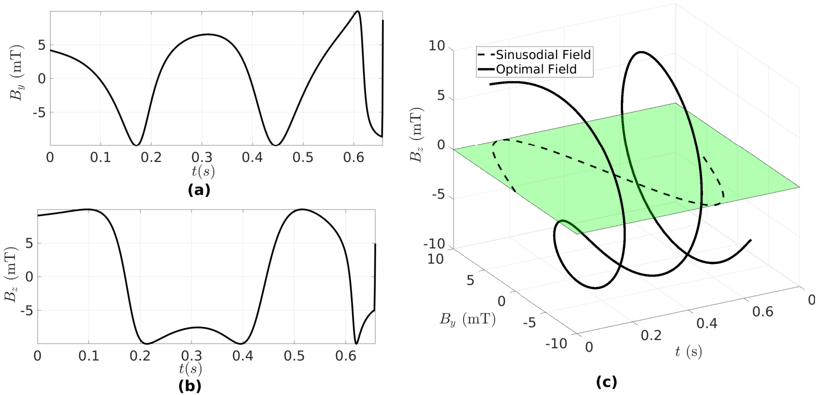
To take into account the fluid-swimmer interaction constrains

Numerical solving and experimental implementation

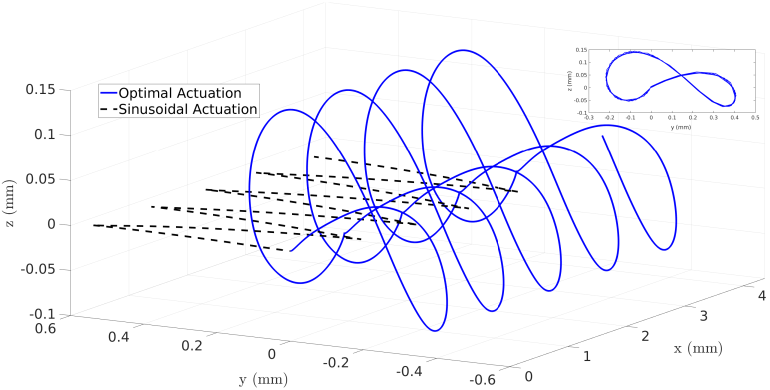
- Direct method
  $\rightarrow$ Discretization of the problem + resolution

- ICLOCS solver + IPOPT for the NLP
  [ICLOCS, P. Falugi , E. Kerrigan , E. Van Wyk, 2010]

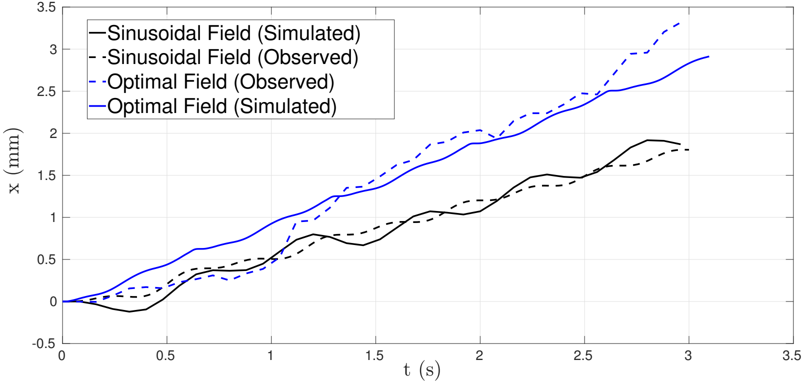
Numerical Solution -Magnetic Fields



Numerical Solution - Trajectory



Experimental Results - Horizontal Displacements



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To further - Challenges

- **Non-planar actuation allows a faster propulsion speed**
- simple RFT-based dynamic model is enough
- How to take into account complex environnement ?
- $\hookrightarrow$ Accurate models
- Navier-Stokes equations
- non-Newtonian fluid

Full Mathematical Model

- Fluid equations

$$
\begin{cases}
\rho(\partial_t u + u \cdot \nabla u) - \mu \Delta u + \nabla p = 0, & \text{in } \mathcal{F}_t, \\
\nabla \cdot u = 0, & \text{in } \mathcal{F}_t, \\
u = U(t), & \text{on } \partial \mathcal{N}_t \ , \\
m\dot{\mathbf{v}} = -F_{fluid} \ , & \\
J\dot{\Omega} = -M_{fluid} \ , &
\end{cases}
$$

where $U := \mathbf{v} + \Omega \times (x - x^{CM}(t)) + u_d(t)$

- Hyper-elasticity equations

$$
\left\{ \rho_s \frac{\partial^2 \eta}{\partial t^2} - \nabla_X \cdot (F\Sigma) = F_{prop} \ , \qquad \text{on } \mathcal{N}_0. \right.
$$

- Coupling

$$
\begin{cases}
\dfrac{\partial \eta}{\partial t} = u_d(t, x), & \text{on } \partial \mathcal{N}_0, \\
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\end{cases}
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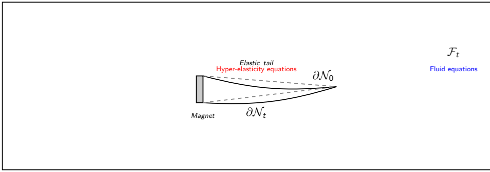
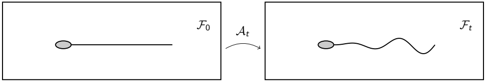


Step I : prescribed deformation - fluid part

$$\begin{cases} \rho(\partial_t u|_{\mathcal{A}} + (u - u_{\mathcal{A}}) \cdot \nabla u) - \mu \Delta u + \nabla p = 0, & \text{in } \mathcal{F}_t, \\ \nabla \cdot u = 0, & \text{in } \mathcal{F}_t, \\ u = U \circ \mathcal{A}_t, & \text{on } \partial \mathcal{N}_t \ , \\ m\dot{\mathbf{v}} = -F_{fluid} \ , & \\ J\dot{\Omega} = -M_{fluid} \ , & \end{cases}$$

$$U := \underbrace{\mathbf{v} + \Omega \times (x - x^{CM}(t))}_{\text{solid motion}} + \overbrace{u_d(t)}^{\text{deformation}}$$

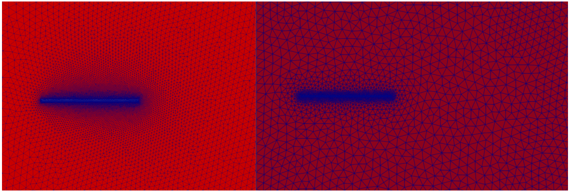


Numerical methods to simulate the swimmer motion

- ODE Solver - RFT
  [Desimone et al. 2015 ...]
  - Impossibility to take into account the complexity of the environnement
  - Impossibility to take into account the complexity of swimmer's shape
- Boundary element methods
  [Pozrikidis 2002, Ishimoto et al. 2016, Alouges et al. 2020 ...]
  - Only for Stokes equations
  - Impossibility to take into account the elasticity of the structure
- Finite element methods
  [A. Iollo et al. 2016, Prud'Homme et al. 2018, ...]

Numerical methods

- Generalizing [Maury, 2000]
- Time discretization
- Spatial discretization → conforming Lagrange finite elements
- Moving domain → Arbitrary-Lagrangian-Eulerian technics
- Mesh (using MMG)
  - Mesh quality indices
  - Re-meshing metric → distance to the swimmer
  - Interpolating
- Feel++



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Algebraic strategy

$$\underbrace{\mathcal{P}^T A \mathcal{P}}_{\mathcal{A}} \begin{bmatrix} \mathbf{u}_I \\ \mathbf{u}_\Gamma \\ \mathbf{U} \\ \omega \\ p \end{bmatrix} = \mathcal{P}^T \begin{bmatrix} G_I \\ G_\Gamma \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

- Requires efficient implementation of $\mathcal{P}^T A \mathcal{P}$ in parallel
- Use a block preconditioner of type PCD or PMM

Results

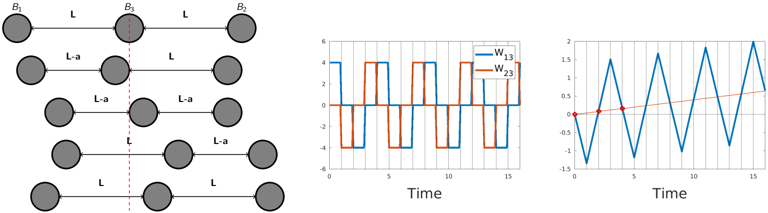
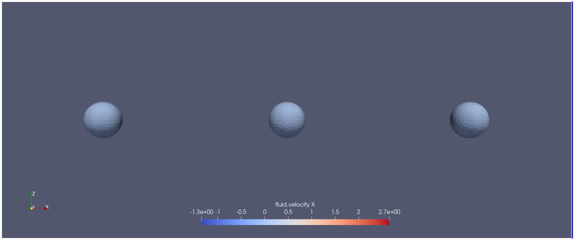


FIGURE: Three-sphere swimmer and its swimming gait.



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Problems

- Hight numerical complexity
- Optimize - control are challenging tasks
- $\rightarrow$ Machine learning tools

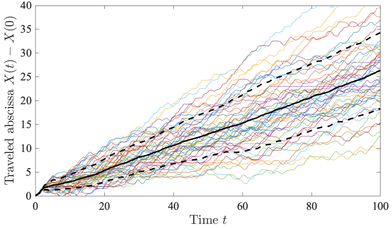
Preliminary work : learn in a turbulent flow

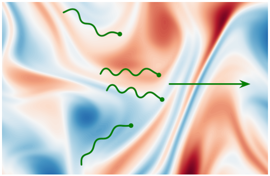
- Cosserat equations for thin inextensible fiber

$$\sigma\,\partial_t^2\mathbf{X} = -\zeta\,\mathbb{R}\,[\partial_t\mathbf{X} - \mathbf{u}(\mathbf{X},t)] + \partial_s(T\partial_s\mathbf{X}) - K\,\partial_s^4\mathbf{X} + \mathbf{f}(s,t).$$

with $\partial_s^2\mathbf{X}(s,t) = 0$ and $T(s,x) = 0$ at the extremities

- Slender body theory [A. Lindner and M. J. Shelley, 2015]
- Reinforcement Learning
- Problem of the chaoticity of the system (fluid-swimmer)





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Thank you for your attention

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