Navigation of Flagellated Micro-Swimmers

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Challenges of micro-swimming

- Medical applications: driving micro-robots through our body
- Multidisciplinary fields (Maths, Physics, Biology and Robotic)
- Biomimeticism
Flagellated locomotion

- Breakthrough $\rightarrow$ in vivo experiments
- Problems: rigid robots
- The hope of the flagellated locomotion?
  - Capability to adapt the strategy
- How to control a flagellated robot?
  - Mathematical modeling
  - Control - optimization tools

[B. Nelson et al., 2015]

[I. S. M. Khalil et al. 2019]
Content

1. Optimization/Control of the magnetic micro-robot
   with Y. El Alaoui Faris, J.-B. Pomet, S. Régnier

2. Accurate simulations for articulated/flagellated swimmer
   with L. Berti, V. Chabannes, C. Prud’Homme

3. Smart swimmer in a turbulent flow
   with J. Bec, R. Chesneaux
Artificial flexible magnetic micro-swimmers

- Imitate a sperm cell propulsion
- Flexible tail + magnetized head
- Magnetic field for producing a flagella-beat waveform
- Length $\sim 7\text{mm}$ - Thickness $\sim 1.5\text{mm}$
  - Highly viscous fluid
  - Low Reynolds Number

Experimental swimmer, ISIR, Sorbonne university, Paris (S. Régnier)
Experimental Setup - Magnetic Field Generation

- 3 orthogonal Helmholtz coils.
- 2 cameras.
Controlling using a sinusoidal field

\[ B_{\parallel} x_{\parallel} + B_{\perp} \sin(2\pi f t) x_{\perp} \]

- Planar displacement (\( x_{\parallel} \)-direction)
- Frequency (\( f \)) \( \Rightarrow \) deformation \( \Rightarrow \) displacement

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[Link]
Challenges

- Symmetric field produces a symmetric deformation
- ⇒ a transversal displacement

Sinusoidal field to control the flexible swimmer

- is not optimized
- does not work in the case of complex geometry / bodily fluid
How to find another magnetic field to drive a magnetic flexible micro-swimmer?

Answer
Optimization problem

Difficulties for predictions
- Hydrodynamic: interaction between fluid and swimmer
- Elasticity: tail’s deformation deriving from the magnetic field
Full Mathematical Model

- Fluid equations

\[
\begin{aligned}
\rho (\partial_t u + u \cdot \nabla u) - \mu \Delta u + \nabla p &= 0, \\
\nabla \cdot u &= 0,
\end{aligned}
\]

in \( \mathcal{F}_t \),

\[
\begin{aligned}
\nabla \cdot u &= 0, \\
u &= U(t),
\end{aligned}
\]

in \( \mathcal{F}_t \),

on \( \partial \mathcal{N}_t \),

\[
m \dot{\mathbf{v}} = - F_{\text{fluid}} ,
\]

\[
J \dot{\Omega} = - M_{\text{fluid}} ,
\]

where \( U := \mathbf{v} + \Omega \times (x - x^{CM}(t)) + u_d(t) \)

- Hyper-elasticity equations

\[
\begin{aligned}
\rho_s \frac{\partial^2 \eta}{\partial t^2} - \nabla \times (F \Sigma) &= F_{\text{prop}} , \\
\end{aligned}
\]

on \( \mathcal{N}_0 \).

- Coupling

\[
\begin{aligned}
\frac{\partial \eta}{\partial t} &= u_d(t, x), \\
F \Sigma n &= \sigma_f n,
\end{aligned}
\]

on \( \partial \mathcal{N}_0 \).
Modeling : main ingredients

▶ Discretization of the shape
- 2D N-link swimmer [Alouges, DeSimone, Giraldi, Zoppello, 2013]

▶ Hydrodynamics (h) : Resistive Force Theory

▶ Elasticity (el) : spring at the junctions

▶ Magnetism (m) : torques which tend to align the magnet
Dynamics

The dynamics of the micro-robot is governed by an ODE linear with respect to the external magnetic field (i.e., control function $B$) with a drift term,

$$
\begin{bmatrix}
\dot{X} \\
\dot{\Theta} \\
\dot{\Phi}
\end{bmatrix} = F_0(\Theta, \Phi) + F_1(\Theta, \Phi)B(t),
$$

Restoring force

Avantage

- Easy to assemble and to compute

Inconvenient

- Lose complexity of surrounding fluid environnement
RFT in good agreement

- Goal: match the propulsion characteristics
  - Velocity-Frequency response to a sinusoidal magnetic field
  - Identification of elastic and hydrodynamical parameters

![Graph showing simulated and experimental mean speeds against frequency](image)
Optimal control problem

- for a given $T$, find $B$ in such a way that the mean speed of the swimmer is maximized for a $x$-displacement

\[
\min -\frac{x(T)}{T}
\]

\[
\begin{bmatrix}
\dot{X} \\
\dot{\Theta} \\
\dot{\Phi}
\end{bmatrix}
= F(\Theta, \Phi, B)
\]

$B$ bounded

orientation and shape $T$-periodic $\leftrightarrow$ stroke constraint

- With this framework, there are many approaches
Several approaches

- Using Pontryaguine principle (or equivalently Lagrange equations)
  [Alouges, Difratta, 2019]
  [Alouges, DeSimone, Giraldi, Or, Weizel, 2019]
  [Loheac and al., 2012]

- Using a direct method
  [Giraldi, Martinon, Zoppello, 2015]
  [Alouges, DeSimone, Heltai, 2013]

- Performing the optimization on specific family of functions
  [Tam, Hosoi, 2007]

Difficulties
To take into account the fluid-swimmer interaction constrains
Numerical solving and experimental implementation

- Direct method
  → Discretization of the problem + resolution

- ICLOCS solver + IPOPT for the NLP
Numerical Solution - Magnetic Fields

(a) $B_y$ vs. $t(s)$

(b) $B_z$ vs. $t(s)$

(c) Comparison of Sinusoidal Field and Optimal Field in 3D space
Numerical Solution - Trajectory
Experimental Results - Horizontal Displacements

![Graph showing experimental results for horizontal displacements with different fields and observed/simulated data.]

- Sinusoidal Field (Simulated)
- Sinusoidal Field (Observed)
- Optimal Field (Observed)
- Optimal Field (Simulated)

Graph axes:
- x (mm)
- t (s)
To further - Challenges

- Non-planar actuation allows a faster propulsion speed
- simple RFT-based dynamic model is enough
- How to take into account complex environment?
  - → Accurate models
  - Navier-Stokes equations
- non-Newtonian fluid
Full Mathematical Model

- Fluid equations

\[
\begin{align*}
\rho \left( \partial_t u + u \cdot \nabla u \right) - \mu \Delta u + \nabla p &= 0, \quad \text{in } \mathcal{F}_t, \\
\nabla \cdot u &= 0, \quad \text{in } \mathcal{F}_t, \\
\nabla \cdot u &= 0, \quad \text{in } \mathcal{F}_t, \\
\n\dot{m} \dot{\mathbf{v}} &= -F_{\text{fluid}}, \\
J \dot{\Omega} &= -M_{\text{fluid}},
\end{align*}
\]

where \( U := \mathbf{v} + \Omega \times (x - x^{CM}(t)) + u_d(t) \)

- Hyper-elasticity equations

\[
\begin{align*}
\rho_s \frac{\partial^2 \eta}{\partial t^2} - \nabla \cdot (F \Sigma) &= F_{\text{prop}}, \quad \text{on } \mathcal{N}_0.
\end{align*}
\]

- Coupling

\[
\begin{align*}
\frac{\partial \eta}{\partial t} &= u_d(t, x), \quad \text{on } \partial \mathcal{N}_0, \\
F \Sigma n &= \sigma_f n, \quad \text{on } \partial \mathcal{N}_0.
\end{align*}
\]
Step 1: prescribed deformation - fluid part

\[
\begin{aligned}
\rho (\partial_t u|_A + (u - u_A) \cdot \nabla u) - \mu \Delta u + \nabla p &= 0, \quad \text{in } \mathcal{F}_t, \\
\nabla \cdot u &= 0, \quad \text{in } \mathcal{F}_t, \\
\nabla \cdot \mathbf{A} &= 0, \quad \text{on } \partial \mathcal{N}_t, \\
\dot{u} &= U \circ \mathcal{A}_t, \\
\dot{m} \mathbf{v} &= -F_{\text{fluid}}, \\
J \dot{\Omega} &= -M_{\text{fluid}},
\end{aligned}
\]

\[U := \mathbf{v} + \Omega \times (\mathbf{x} - \mathbf{x}^{CM}(t)) + \widehat{u_d}(t)\]
Numerical methods to simulate the swimmer motion

- **ODE Solver - RFT**
  - [Desimone et al. 2015 ...]
    - Impossibility to take into account the complexity of the environnement
    - Impossibility to take into account the complexity of swimmer’s shape

- **Boundary element methods**
    - Only for Stokes equations
    - Impossibility to take into account the elasticity of the structure

- **Finite element methods**
  - [A. Iollo et al. 2016, Prud’Homme et al. 2018, ...]
Numerical methods

- Generalizing [Maury, 2000]
- Time discretization
- Spatial discretization $\rightarrow$ conforming Lagrange finite elements
- Moving domain $\rightarrow$ Arbitrary-Lagrangian-Eulerian technics
- Mesh (using MMG)
  - Mesh quality indices
  - Re-meshing metric $\rightarrow$ distance to the swimmer
  - Interpolating
- Feel++
Algebraic strategy

\[ \mathcal{P}^T A \mathcal{P} \begin{bmatrix} u_l \\ u_\Gamma \\ U \\ \omega \\ \rho \end{bmatrix} = \mathcal{P}^T \begin{bmatrix} G_l \\ G_\Gamma \\ 0 \\ 0 \\ 0 \end{bmatrix}. \]

- Requires efficient implementation of \( \mathcal{P}^T A \mathcal{P} \) in parallel
- Use a block preconditioner of type PCD or PMM


**Results**

\[ B_1 \quad L \quad B_2 \quad L \quad B_3 \quad L - a \quad L - a \quad L - a \quad L \quad L - a \quad L \]

**Figure:** Three-sphere swimmer and its swimming gait.
Problems

- Hight numerical complexity
- Optimize - control are challenging tasks
- \(\rightarrow\) Machine learning tools
Preliminary work: learn in a turbulent flow

- Cosserat equations for thin inextensible fiber

\[
\sigma \partial_t^2 \mathbf{X} = -\zeta \mathbb{R} \left[ \partial_t \mathbf{X} - \mathbf{u}(\mathbf{X}, t) \right] \\
+ \partial_s \left( T \partial_s \mathbf{X} \right) - K \partial_s^4 \mathbf{X} + \mathbf{f}(s, t).
\]

with \( \partial_s^2 \mathbf{X}(s, t) = 0 \) and \( T(s, x) = 0 \) at the extremities

- Slender body theory [A. Lindner and M. J. Shelley, 2015]

- Reinforcement Learning

- Problem of the chaoticity of the system (fluid-swimmer)
Thank you for your attention

https://www-sop.inria.fr/members/Laetitia.Giraldi/