EPIDEMIC CONTROL THROUGH INCENTIVES, LOCKDOWN, AND TESTING

The government’s perspective

Emma HUBERT¹

¹Department of Mathematics & CFM, Imperial College London
1. Introduction
   - Motivation: epidemic control
   - Incentives and contract theory

2. The model: SIR and principal-agent
   - Epidemic model
   - Principal-agent problem

3. Resolution
   - Relevant tax policy
   - Numerical results

4. Limits and extensions

5. Bibliography
INTRODUCTION
Impact of the epidemic. Cost for the society:

- cost of care related to the disease;
- indirect costs related to the saturation of the hospital system;
- but also a cost for the individual, not necessarily monetary, linked to the infection (QALY/DALY).

► How to control this epidemic, in order to limit these costs?
Impact of the epidemic. Cost for the society:

- cost of care related to the disease;
- indirect costs related to the saturation of the hospital system;
- but also a cost for the individual, not necessarily monetary, linked to the infection (QALY/DALY).

How to control this epidemic, in order to limit these costs?

Lockdown and social distancing. While waiting for:

- an effective treatment against the disease induced by the virus;
- a vaccination campaign...

Many countries have implemented restrictions on travel and social distancing between individuals to limit the spread of the epidemic.
Impact of the epidemic. Cost for the society:

- cost of care related to the disease;
- indirect costs related to the saturation of the hospital system;
- but also a cost for the individual, not necessarily monetary, linked to the infection (QALY/DALY).

▶ How to control this epidemic, in order to limit these costs?

Lockdown and social distancing. While waiting for:

- an effective treatment against the disease induced by the virus;
- a vaccination campaign...

▶ Many countries have implemented restrictions on travel and social distancing between individuals to limit the spread of the epidemic.

Impact of social distancing. These restrictions have a cost for individuals: social and economic, but also in terms of (mental) health (no in-person conferences at La Grande Motte).
DIFFERENT POINTS OF VIEW

Epidemic control via social distancing: political or individual choice?
Epidemic control via social distancing: political or individual choice?


Without guidelines? Individual point of view, with R. Elie and G. Turinici.

Modeling individual choices towards the epidemic spread, and look for a Nash – Mean-field equilibrium among the population.

Cost of anarchy: Nash equilibrium different from societal optimum...

How can we make the interests of the population converge towards the interests of the society?

▶ Governmental point of view and incentives, with T. Mastrolia, D. Possamaï and X. Warin.
Epidemic control via social distancing: political or individual choice?


▶ Without guidelines? Individual point of view, with R. Elie and G. Turinici.
▶ Modeling individual choices towards the epidemic spread, and look for a Nash – Mean-field equilibrium among the population.
Epidemic control via social distancing: political or individual choice?

► **Societal Optimum.** Recent literature focuses on the search for a societal optimum – Viewpoint of a *global planner*. Bonnans and Gianatti \[2\] (2020), Charpentier et al. \[3\] (2020), Djidjou-Demasse et al. \[4\] (2020)...


► Modeling individual choices towards the epidemic spread, and look for a Nash – Mean-field equilibrium among the population.

► Cost of anarchy: Nash equilibrium different from societal optimum...

How can we make the interests of the population converge towards the interests of the society?

► **Governmental point of view and incentives**, with T. Mastrolia, D. Possamaï and X. Warin.

- Analyse interactions between economic agents, in particular with asymmetric information.

▶ Analyse interactions between economic agents, in particular with asymmetric information.

The Principal (she) initiates a contract for a period $[0, T]$.

The Agent (he) accepts or not the contract proposed by the Principal.

- Analyse interactions between economic agents, in particular with asymmetric information.

The Principal (she) initiates a contract for a period \([0, T]\).

The Agent (he) accepts or not the contract proposed by the Principal.

The Principal must suggest an optimal contract: maximises her utility, and that the Agent will accept (reservation utility).

- Analyse interactions between economic agents, in particular with asymmetric information.

The Principal (she) initiates a contract for a period \([0, T]\).

The Agent (he) accepts or not the contract proposed by the Principal.

The Principal must suggest an optimal contract: maximises her utility, and that the Agent will accept (reservation utility).

Asymmetry of information.

Moral Hazard: the Agent’s behaviour is not observable by the Principal.
Output process: Stochastic process $X$ with dynamic, for $t \in [0, T]$:
\[ dX_t = \alpha_t dt + \sigma_t dW_t. \]

Effort: given a contract $\xi$, the Agent controls $X$ through the drift $\alpha$, in order to maximise the following criteria:
\[
\mathbb{E}_{\mathbb{P}^\alpha} \left[ U_A(\xi) - \int_0^T c(\alpha_t) dt \right].
\]

Moral Hazard: the Principal only observes $X$ in continuous-time.
Output process: Stochastic process $X$ with dynamic, for $t \in [0, T]$:

$$dX_t = \alpha_t dt + \sigma_t dW_t.$$ 

Effort: given a contract $\xi$, the Agent controls $X$ through the drift $\alpha$, in order to maximise the following criteria:

$$\mathbb{E}^{p,\alpha} \left[ U_A(\xi) - \int_0^T c(\alpha_t) dt \right].$$

Moral Hazard: the Principal only observes $X$ in continuous-time.

- The contract (terminal payment) $\xi$ can only be indexed on $X$. 
Output process: Stochastic process $X$ with dynamic, for $t \in [0, T]$:

$$dX_t = \alpha_t \, dt + \sigma_t \, dW_t.$$  

Effort: given a contract $\xi$, the Agent controls $X$ through the **drift** $\alpha$, in order to maximise the following criteria:

$$\mathbb{E}^{\mathbb{P}_\alpha} \left[ U_A(\xi) - \int_0^T c(\alpha_t) \, dt \right].$$

Moral Hazard: the Principal only observes $X$ in continuous-time.

- The contract (terminal payment) $\xi$ can only be indexed on $X$.
- The **optimal** form of contracts for the Agent satisfies (see [5, 6]):

$$U_A(\xi) = Y_0 - \int_0^T \mathcal{H}(Z_s) \, ds + \int_0^T Z_s \, dX_s,$$  

(1)
**Output process:** Stochastic process $X$ with dynamic, for $t \in [0, T]$:

$$dX_t = \alpha_t dt + \sigma_t dW_t.$$

**Effort:** given a contract $\xi$, the Agent controls $X$ through the drift $\alpha$, in order to maximise the following criteria:

$$\mathbb{E}^{p, \alpha}[U_A(\xi) - \int_0^T c(\alpha_t)dt].$$

**Moral Hazard:** the Principal only observes $X$ in continuous-time.

- The contract (terminal payment) $\xi$ can only be indexed on $X$.
- The optimal form of contracts for the Agent satisfies (see [5, 6]):

$$U_A(\xi) = Y_0 - \int_0^T \mathcal{H}(Z_s)ds + \int_0^T Z_s dX_s,$$

where

(i) $Y_0 \in \mathbb{R}$ is chosen by the principal to ensure participation of the agent;
(ii) $Z$ is chosen by the Principal to encourage effort from the agent;
(iii) $\mathcal{H}$ is the Agent’s Hamiltonian.
THE MODEL: SIR AND PRINCIPAL-AGENT
SIR compartment model: during the epidemic, individuals go from “Susceptible” to “Infected” and then “Recovered”.

Figure: SIR model
EPIDEMIC MODEL

▶ SIR compartment model: during the epidemic, individuals go from “Susceptible” to “Infected” and then “Recovered”.

![SIR model diagram](image)

**Figure:** SIR model

▶ Dynamic of a stochastic SIR model:

\[
\begin{aligned}
    dS_t &= -\beta_t \sqrt{\alpha_t S_t I_t} dt + \sigma \alpha_t S_t I_t dW_t, \\
    dl_t &= (\beta_t \sqrt{\alpha_t S_t} - \gamma)l_t dt - \sigma \alpha_t S_t I_t dW_t, \quad \text{for } t \in [0, T], \\
    dR_t &= \gamma l_t dt,
\end{aligned}
\]
EPIDEMIC MODEL

▶ SIR compartment model: during the epidemic, individuals go from “Susceptible” to “Infected” and then “Recovered”.

Figure: SIR model

▶ Dynamic of a stochastic SIR model:

\[
\begin{align*}
    dS_t &= -\beta_t \sqrt{\alpha_t S_t I_t} dt + \sigma \alpha_t S_t I_t dW_t, \\
    dl_t &= (\beta_t \sqrt{\alpha_t S_t} - \gamma) I_t dt - \sigma \alpha_t S_t I_t dW_t, \quad \text{for } t \in [0, T]. \\
    dR_t &= \gamma I_t dt,
\end{align*}
\]

Initial distribution \((S_0, I_0, R_0)\) at time \(t = 0\) known, s.t. \(S_0 + I_0 + R_0 = 1\).

▶ \(S_t + I_t + R_t = 1\) for all \(t \geq 0\).

▶ 3 main parameters to describe the dynamics of the epidemic: \(\gamma\), \(\beta\) and \(\alpha\).
Recovery rate $\gamma$. Exogenous, constant, assumed known, and given by

$$\gamma := \frac{1}{\text{duration of the contagious period}}.$$ 

In absence of an effective treatment against the disease induced by the virus, no possibility to control $\gamma$. 

\[
\begin{align*}
    dS_t &= -\beta_t \sqrt{\alpha_t} S_t I_t dt + \sigma \alpha_t S_t I_t W_t, \\
    dl_t &= (\beta_t \sqrt{\alpha_t} S_t - \gamma) I_t dt - \sigma \alpha_t S_t I_t W_t, \quad \text{for } t \in [0, T], \\
    dR_t &= \gamma I_t dt,
\end{align*}
\]
PARAMETERS OF THE EPIDEMIC: TRANSMISSION RATE

\[
\begin{cases}
  dS_t = -\beta_t \sqrt{\alpha_t} S_t I_t dt + \sigma \alpha_t S_t I_t dW_t, \\
  dl_t = (\beta_t \sqrt{\alpha_t} S_t - \gamma) I_t dt - \sigma \alpha_t S_t I_t dW_t, \quad \text{for } t \in [0, T], \\
  dR_t = \gamma I_t dt,
\end{cases}
\]

▶ **Transmission rate \( \beta \)**. Endogenous, and time-dependent (in contrast to the classical SIR models).
PARAMETERS OF THE EPIDEMIC: TRANSMISSION RATE

\[
\begin{align*}
\text{d}S_t &= -\beta_t \sqrt{\alpha_t} S_t I_t \, dt + \sigma \alpha_t S_t I_t \, dW_t, \\
\text{d}I_t &= (\beta_t \sqrt{\alpha_t} S_t - \gamma) I_t \, dt - \sigma \alpha_t S_t I_t \, dW_t, \quad \text{for } t \in [0, T], \\
\text{d}R_t &= \gamma I_t \, dt,
\end{align*}
\]

► Transmission rate \( \beta \). Endogenous, and time-dependent (in contrast to the classical SIR models). Depends on:

(i) intrinsic characteristics of the disease;
(ii) “contact rate” between individuals.
PARAMETERS OF THE EPIDEMIC: TRANSMISSION RATE

\[
\begin{align*}
\text{d}S_t &= -\beta_t \sqrt{\alpha_t} S_t I_t \text{d}t + \sigma \alpha_t S_t I_t \text{d}W_t, \\
\text{d}I_t &= (\beta_t \sqrt{\alpha_t} S_t - \gamma) I_t \text{d}t - \sigma \alpha_t S_t I_t \text{d}W_t, \quad \text{for } t \in [0, T], \\
\text{d}R_t &= \gamma I_t \text{d}t,
\end{align*}
\]

- **Transmission rate** $\beta$. Endogenous, and time-dependent (in contrast to the classical SIR models). Depends on:
  
  (i) intrinsic characteristics of the disease;
  (ii) “contact rate” between individuals.

- The population can make a **costly effort** to reduce their interactions and thus decrease the effective transmission rate of the virus.
PARAMETERS OF THE EPIDEMIC: TRANSMISSION RATE

\[
\begin{align*}
\frac{dS_t}{dt} &= -\beta_t \sqrt{S_t I_t} dt + \sigma \alpha_t S_t I_t dW_t, \\
\frac{dI_t}{dt} &= (\beta_t \sqrt{S_t} - \gamma) I_t dt - \sigma \alpha_t S_t I_t dW_t, \quad \text{for } t \in [0, T]. \\
\frac{dR_t}{dt} &= \gamma I_t dt,
\end{align*}
\]

▶ Transmission rate \( \beta \). Endogenous, and time-dependent (in contrast to the classical SIR models). Depends on:

(i) intrinsic characteristics of the disease;
(ii) “contact rate” between individuals.

▶ The population can make a costly effort to reduce their interactions and thus decrease the effective transmission rate of the virus.

▶ We assume that \( \beta \) takes values in \([0, \beta_0]\), where \( \beta_0 \) is the “initial” transmission rate, when the population makes no social distancing effort.

▶ Limiting case: if individuals are fully isolated, then \( \beta = 0 \) and the virus does not spread.
PARAMETERS OF THE EPIDEMIC: “UNCERTAINTY RATE”

\[
\begin{align*}
    dS_t &= -\beta_t \sqrt{\alpha_t} S_t I_t dt + \sigma \alpha_t S_t I_t dW_t, \\
    dl_t &= (\beta_t \sqrt{\alpha_t} S_t - \gamma) I_t dt - \sigma \alpha_t S_t I_t dW_t, \quad \text{for } t \in [0, T]. \\
    dR_t &= \gamma I_t dt,
\end{align*}
\]

▶ “Uncertainty rate” $\alpha$. By increasing testing among the population, the government can:

(i) to reduce the uncertainty related to the spread of the epidemic, i.e. $\sigma \alpha_t$;

(ii) to isolate those who test positive and thus reduce the effective spread rate, i.e. $\beta_t \sqrt{\alpha_t}$. 

By increasing testing among the population, the government can:

(i) to reduce the uncertainty related to the spread of the epidemic, i.e. $\sigma \alpha_t$;

(ii) to isolate those who test positive and thus reduce the effective spread rate, i.e. $\beta_t \sqrt{\alpha_t}$. 

We assume that $\alpha$ takes values in $[0, 1]$:

• $\alpha = 1$ means no testing policy;

• limiting case: if all individuals are tested regularly, the spread of the epidemic is precisely known (without randomness), and isolation of positive individuals stop the propagation, i.e. $\alpha \to 0$. 

11
PARAMETERS OF THE EPIDEMIC: “UNCERTAINTY RATE”

\[
\begin{align*}
    dS_t &= -\beta_t \sqrt{\alpha_t} S_t I_t dt + \sigma \alpha_t S_t I_t dW_t, \\
    dl_t &= (\beta_t \sqrt{\alpha_t} S_t - \gamma) l_t dt - \sigma \alpha_t S_t l_t dW_t, \quad \text{for } t \in [0, T], \\
    dR_t &= \gamma l_t dt,
\end{align*}
\]

▸ “Uncertainty rate” \( \alpha \). By increasing testing among the population, the government can:

(i) to reduce the uncertainty related to the spread of the epidemic, i.e. \( \sigma \alpha_t \);
(ii) to isolate those who test positive and thus reduce the effective spread rate, i.e. \( \beta_t \sqrt{\alpha_t} \).

▸ We assume that \( \alpha \) takes values in \([0, 1] \):

- \( \alpha = 1 \) means no testing policy;
- limiting case: if all individuals are tested regularly, the spread of the epidemic is precisely known (without randomness), and isolation of positive individuals stop the propagation, i.e. \( \alpha \rightarrow 0 \).
The population – the agent – controls the transmission rate $\beta$. 

The government – the principal – observes $(S, I, R)$ (not $\beta$) and implements two policies:

(i) a testing policy $\alpha$, which enables
   • to reduce the uncertainty related to the spread of the epidemic;
   • to isolate those who test positive and thus reduce the effective spread rate;

(ii) a tax policy $\chi$ to encourage the population to lockdown.

Stackelberg equilibrium.

(i) Given a pair $(\alpha, \chi)$ fixed by the government, solve the population's optimisation problem to find the optimal transmission rate $\beta^\star(\alpha, \chi)$.

(ii) Given the optimal response of the population to any couple $(\chi, \alpha)$, solve the government's problem to find the optimal couple $(\chi^\star, \alpha^\star)$. 

12
The population – the agent – controls the transmission rate $\beta$.

The government – the principal – observes $(S, I, R)$ (not $\beta$) and implements two policies:

(i) a testing policy $\alpha$, which enables
  - to reduce the uncertainty related to the spread of the epidemic;
  - to isolate those who test positive and thus reduce the effective spread rate;

(ii) Stackelberg equilibrium.

(i) Given a pair $(\alpha, \chi)$ fixed by the government, solve the population’s optimisation problem to find the optimal transmission rate $\beta^\star(\alpha, \chi)$.

(ii) Given the optimal response of the population to any couple $(\chi, \alpha)$, solve the government’s problem to find the optimal couple $(\chi^\star, \alpha^\star)$.
The population – the agent – controls the transmission rate $\beta$.

The government – the principal – observes $(S, I, R)$ (not $\beta$) and implements two policies:

(i) a testing policy $\alpha$, which enables
   - to reduce the uncertainty related to the spread of the epidemic;
   - to isolate those who test positive and thus reduce the effective spread rate;

(ii) a tax policy $\chi$ to encourage the population to lockdown.
The population – the agent – controls the transmission rate $\beta$.

The government – the principal – observes $(S, I, R)$ (not $\beta$) and implements two policies:

(i) a testing policy $\alpha$, which enables
   - to reduce the uncertainty related to the spread of the epidemic;
   - to isolate those who test positive and thus reduce the effective spread rate;

(ii) a tax policy $\chi$ to encourage the population to lockdown.

Stackelberg equilibrium.

(i) Given a pair $(\alpha, \chi)$ fixed by the government, solve the population’s optimisation problem to find the optimal transmission rate $\beta^*(\alpha, \chi)$. 
The population – the agent – controls the transmission rate $\beta$.

The government – the principal – observes $(S, I, R)$ (not $\beta$) and implements two policies:

(i) a testing policy $\alpha$, which enables
   - to reduce the uncertainty related to the spread of the epidemic;
   - to isolate those who test positive and thus reduce the effective spread rate;

(ii) a tax policy $\chi$ to encourage the population to lockdown.

Stackelberg equilibrium.

(i) Given a pair $(\alpha, \chi)$ fixed by the government, solve the population’s optimisation problem to find the optimal transmission rate $\beta^*(\alpha, \chi)$.

(ii) Given the optimal response of the population to any couple $(\chi, \alpha)$, solve the government’s problem to find the optimal couple $(\chi^*, \alpha^*)$. 

RESOLUTION
Given a testing policy $\alpha$ and a tax policy $\chi$, the population’s optimal control problem is:

$$V_0^A(\alpha, \chi) := \sup_{\beta \in B} \mathbb{E} \left[ \int_0^T u(t, \beta_t, I_t) dt + U(-\chi) \right],$$

(2)
Given a testing policy $\alpha$ and a tax policy $\chi$, the population’s optimal control problem is:

$$V_0^A(\alpha, \chi) := \sup_{\beta \in \mathcal{B}} \mathbb{E} \left[ \int_0^T u(t, \beta_t, I_t)dt + U(-\chi) \right],$$

(2)

where $u : [0, T] \times [0, \beta_0] \times \mathbb{R}_+ \rightarrow \mathbb{R}$ and $U : \mathbb{R} \rightarrow \mathbb{R}$ are given by:

$$u(t, b, i) := -\frac{1}{2}(i^3 + (\beta_0 - b)^2) \text{ and } U(x) := \frac{1 - e^{-4x}}{4} + \frac{1}{2}x.$$
Given a testing policy $\alpha$ and a tax policy $\chi$, the population’s optimal control problem is:

$$V_0^A(\alpha, \chi) := \sup_{\beta \in B} \mathbb{E}\left[\int_0^T u(t, \beta_t, I_t)dt + U(-\chi)\right], \quad (2)$$

where $u : [0, T] \times [0, \beta_0] \times \mathbb{R}^+ \rightarrow \mathbb{R}$ and $U : \mathbb{R} \rightarrow \mathbb{R}$ are given by:

$$u(t, b, i) := -\frac{1}{2}(i^3 + (\beta_0 - b)^2) \quad \text{and} \quad U(x) := \frac{1 - e^{-4x}}{4} + \frac{1}{2}x.$$ 

Interpretations:

(i) the utility is zero when there is no epidemic, i.e. $I = 0$;

(ii) the population is scared by a large number of infected – term $i^3$ in $u$;

(iii) an increase in the tax lowers the utility – $U$ increasing function;

(iv) a decrease in the level of interaction (below $\beta_0$) lowers the utility – $u$ increasing w.r.t. $b \leq \beta_0$. 

Main theoretical result. Given an admissible contract \((\alpha, \chi)\), there exist a unique \(Y_0\) and \(Z\) such that:

\[
U(\chi) = Y_0 \int_T^0 \left( \gamma Z_t I_t + u(t, \beta^\star t, I_t) \beta^\star t \right) \alpha_t S_t I_t Z_t dt \int_T^0 Z_t dt I_t,
\]

where \(\beta^\star\) is the (unique) optimal contact rate for the population.

Optimal effort. For all \(t \in [0, T]\), \(\beta^\star t := b^\star(t, S_t, I_t, Z_t, \alpha_t)\) is the maximiser of:

\[
b^2_0 [0, \beta^\star ]7! u(t, b, I_t) \beta p \alpha_t S_t I_t Z_t.
\]

Under some assumptions for existence and smoothness of the inverse of the function \(U\), (3) gives a representation for the tax \(\chi\).
Main theoretical result. Given an admissible contract \((\alpha, \chi)\), there exist a unique \(Y_0\) and \(Z\) such that:

\[
U(-\chi) = Y_0 - \int_0^T \left( \gamma Z_t I_t + u(t, \beta^*_t, I_t) - \beta^*_t \sqrt{\alpha_t S_t I_t} Z_t \right) dt - \int_0^T Z_t dl_t, \tag{3}
\]

where \(\beta^*\) is the (unique) optimal contact rate for the population.
HJB technics and BSDEs theory...

**Main theoretical result.** Given an admissible contract \((\alpha, \chi)\), there exist a unique \(Y_0\) and \(Z\) such that:

\[
U(-\chi) = Y_0 - \int_0^T \left( \gamma Z_t l_t + u(t, \beta^*_t, I_t) - \beta^*_t \sqrt{\alpha_t S_t l_t Z_t} \right) dt - \int_0^T Z_t dl_t,
\]

where \(\beta^*\) is the (unique) optimal contact rate for the population.

**Optimal effort.** For all \(t \in [0, T]\), \(\beta^*_t := b^*(t, S_t, I_t, Z_t, \alpha_t)\) is the maximiser of:

\[
b \in [0, \beta_0] \mapsto u(t, b, l_t) - b \sqrt{\alpha_t S_t l_t Z_t}.
\]
Main theoretical result. Given an admissible contract \((\alpha, \chi)\), there exist a unique \(Y_0\) and \(Z\) such that:

\[
U(-\chi) = Y_0 - \int_0^T \left( \gamma Z_t l_t + u(t, \beta_t^*, I_t) - \beta_t^* \sqrt{\alpha_t S_t I_t Z_t} \right) dt - \int_0^T Z_t dl_t,
\]

where \(\beta^*\) is the (unique) optimal contact rate for the population.

Optimal effort. For all \(t \in [0, T]\), \(\beta_t^* := b^*(t, S_t, I_t, Z_t, \alpha_t)\) is the maximiser of:

\[
b \in [0, \beta_0] \mapsto u(t, b, I_t) - b \sqrt{\alpha_t S_t I_t Z_t}.
\]

Under some assumptions for existence and smoothness of the inverse of the function \(U\), (3) gives a representation for the tax \(\chi\).
Given a tax $\chi$ and a testing policy $\alpha$ chosen by the government:
Given a tax $\chi$ and a testing policy $\alpha$ chosen by the government:

- the optimal form for the tax $\chi$ satisfies (3), i.e., the government only has to choose $Z$ and $Y_0$;
Given a tax $\chi$ and a testing policy $\alpha$ chosen by the government:

- the optimal form for the tax $\chi$ satisfies (3), i.e., the government only has to choose $Z$ and $Y_0$;
- the optimal effort of the population is given by $\beta^*_t := b^*(t, S_t, I_t, Z_t, \alpha_t)$ for all $t \in [0, T]$, and thus the epidemic spreads with the transmission rate $\beta^* \sqrt{\alpha}$.

It remains to solve the government problem to find the optimal $Y_0$, $Z$ and $\alpha$. 
The government chooses the parameter $Y_0$ and $Z$ in the tax $\chi$, as well as $\alpha$ to maximise her utility:

$$V^P_0 := \sup_{Y_0, Z, \alpha} \mathbb{E}\left[\chi - \int_0^T (c(l_t) + k(\alpha_t)) \, dt\right],$$

(4)

where $k(a) := \kappa_g (a^{-\eta_g} - 1)$, and $c(i) := c_g (i + i^2)$. 
The government chooses the parameter $Y_0$ and $Z$ in the tax $\chi$, as well as $\alpha$ to maximise her utility:

$$V^P_0 := \sup_{Y_0,Z,\alpha} \mathbb{E} \left[ \chi - \int_0^T (c(I_t) + k(\alpha_t)) \, dt \right],$$

where $k(a) := \kappa_a(a^{-\eta_a} - 1)$, and $c(i) := c_g(i + i^2)$.

Interpretations:

(i) the utility is zero when there is no epidemic, i.e. $I_0 = 0$;
(ii) an increase in the tax increase the utility;
(iii) the principal pay a linear cost per infected individual, plus a quadratic cost to represent saturation of health care facilities;
(iv) testing is costly – $k$ increases when $\alpha$ decreases.
In contrast to usual PA problems, the government implements a mandatory tax: the population cannot refuse it.

Nevertheless, the government is “benevolent”: she will choose $Y_0$ in order to ensure a sufficient level $\underline{v}$ of utility for the agent.

In particular, $\underline{v}$ is defined by the agent’s utility in the event of an uncontrolled epidemic, i.e., $\beta = \beta_0$, $\alpha = 1$, and $\chi = 0$. 

Results.

Theoretically: PDE obtained through HJB technics.

In contrast to usual PA problems, the government implements a mandatory tax: the population cannot refuse it.

Nevertheless, the government is “benevolent”: she will choose $Y_0$ in order to ensure a sufficient level $v$ of utility for the agent.

In particular, $v$ is defined by the agent’s utility in the event of an uncontrolled epidemic, i.e., $\beta = \beta_0$, $\alpha = 1$, and $\chi = 0$.

Results.

- Theoretically: PDE obtained through HJB technics.
SOME NUMERICAL RESULTS: PROPORTION OF INFECTED INDIVIDUALS

Without governmental intervention

With incentives but without testing

With incentives and testing

Without moral hazard (first-best)
LIMITS AND EXTENSIONS
On the epidemic modelling.

- Uncertainty about the parameters, especially when the epidemic is of a new kind.
- Viability of SIR, SEIR models? COVID-19 has many other features, for e.g. large number of asymptomatic.
On the epidemic modelling.
► Uncertainty about the parameters, especially when the epidemic is of a new kind.
► Viability of SIR, SEIR models? COVID-19 has many other features, for e.g. large number of asymptomatic.

On the population’s side.
► Rational population, perfect knowledge of the dynamics of the epidemic...
► Individual’s costs are hard to measure and calibrate.
LIMITS AND EXTENSIONS

On the epidemic modelling.
▶ Uncertainty about the parameters, especially when the epidemic is of a new kind.
▶ Viability of SIR, SEIR models? COVID-19 has many other features, for e.g. large number of asymptomatic.

On the population’s side.
▶ Rational population, perfect knowledge of the dynamics of the epidemic...
▶ Individual’s costs are hard to measure and calibrate.

On the government’s side.
▶ Better assess the costs faced by states: hospital saturation costs, economic cost of lockdown...
On the epidemic modelling.
▶ Uncertainty about the parameters, especially when the epidemic is of a new kind.
▶ Viability of SIR, SEIR models? COVID-19 has many other features, for e.g. large number of asymptomatic.

On the population’s side.
▶ Rational population, perfect knowledge of the dynamics of the epidemic...
▶ Individual’s costs are hard to measure and calibrate.

On the government’s side.
▶ Better assess the costs faced by states: hospital saturation costs, economic cost of lockdown...

Extensions.
• Model by Aurell et al. [1] (2020) that combines MFG in the population and incentives.
• What about vaccination?
BIBLIOGRAPHY


